

# Review of the use of metamaterials for acoustic scattering reduction

Marco Norambuena, Martin Ochmann

*Beuth Hochschule für Technik Berlin, Research Group Computational Acoustics, Berlin, Germany,  
Email: mnorambuena@beuth-hochschule.de, ochmann@beuth-hochschule.de*

## Introduction

In the last 10 years physics has experienced the arrival and growth of a new research field. This new area, called metamaterials, has attracted special interest due to its potential applications. The term metamaterial is a broad definition that describes a group of engineered materials which provide certain properties not commonly available in nature, such as negative refractive index, negative effective mass density, etc. Their particular properties are generated by its internal physical structure rather than its chemical composition.

Although these materials were initially proposed and investigated in the field of electromagnetism, their physical characteristics have allowed to expand their development to other areas such as optics, mechanics, acoustics, etc. where also materials interact with propagating waves. The range where a metamaterial behaves as such occurs where the internal structure of the material is much smaller than the wavelength of the interacting wave.

The incidence of any type of wave on a rigid body, in this case acoustic waves, generates a reflected wave as a result. The control of these reflected waves is particularly important since it allows to improve the acoustic quality in many situations. For example, in architectural acoustics such control of the reflection it translates into a better perception of music or speech. In industrial applications, the use of specially crafted materials as the ones investigated here, allows to reduce the scattering generated by rigid bodies.

## Acoustic cloaking

Cloaking refers to the ability of a metamaterial to make an object invisible, this idea was introduced by Pendry et al. [1,2] in 2006 and since then it has led to numerous papers. The principle of cloaking is based on the form-invariance of Maxwell's equations and a design technique called transformation optics [3,4], which together define how to shape a material to achieve certain purpose, in this case a cloaking device.

In 2007 Cummer et al. [5] presented one of the first acoustic cloaks, this was based on the experience gained through previous work regarding electromagnetic cloak. It exploits the equivalence between acoustic wave propagation in two dimensions and electromagnetic propagation in isotropic media. The duality of both equations is given by the values shown below

$$[p, v_r, v_\phi, \rho_r, \rho_\phi, \lambda^{-1}] \leftrightarrow [-E_z, H_\phi, -H_r, \mu_\phi, \mu_r, \epsilon_z]$$

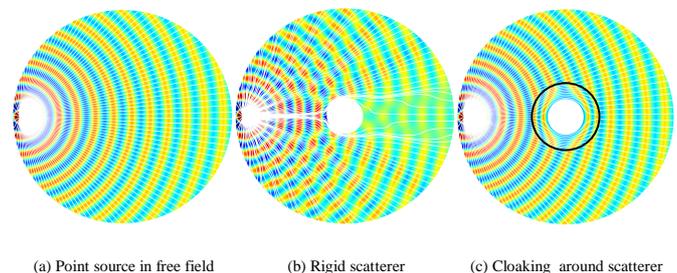
where

| Acoustics       |              | Electromagnetism |                |
|-----------------|--------------|------------------|----------------|
| $p$             | Pressure     | $E_z$            | Electric field |
| $v_{r,\phi}$    | Velocity     | $H_{\phi,r}$     | Magnetic field |
| $\rho_{r,\phi}$ | Mass density | $\mu_{\phi,r}$   | Permeability   |
| $\lambda$       | Bulk modulus | $\epsilon_z$     | Permittivity   |

The result of this equivalence implies that is possible to create a cylindrical shell with a particular spatial distribution of mass density and bulk modulus that will bend the trajectory of any incident wave around the center of a rigid object with minimum scattering, as shown in Fig. 1. The material spatial distribution required for such result is given by

$$\frac{\rho_r}{\rho_0} = \frac{r}{r-a}, \quad \frac{\rho_\phi}{\rho_0} = \frac{r-a}{r}, \quad \frac{\lambda}{\lambda_0} = \left(\frac{b-a}{b}\right)^2 \frac{r}{r-a}$$

where  $r$  is the radius,  $a$  and  $b$  are the inner and outer radii of the cloak, and the quantities with subscript  $0$  are those of the background medium.



**Figure 1:** Sound pressure field under three different conditions.

A year later two new designs for acoustic cloaking were proposed by Cheng et al. [6] and Chen et al. [7], these designs were based on a bi-layered structure that could potentially facilitate the actual realization.

Although the previously proposed cloaking devices proved to greatly minimize the scattering of a reflecting body and therefore provide a reasonable performance, these designs assume that the constitutive materials of the cloak are fluids. However, any practical cloaking would require some sort of solid components to at least sustain its structural integrity. Since the beginning of research in this area it was pointed out that equations of motion for elastic media do not share the form-invariance present in equations of electromagnetism and acoustics.

| Cheng's cloak                                                                                                                                                                                                                                                                          | Chen's cloak                                                                                                                                                                                                                                                                                                     |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\frac{\rho_{2i-1}}{\rho_0} = \frac{r_{2i-1}}{r_{2i-1}-a} - \sqrt{\left(\frac{r_{2i-1}}{r_{2i-1}-a}\right)^2 - 1}$ $\frac{\rho_{2i}}{\rho_0} = \frac{r_{2i}}{r_{2i}-a} + \sqrt{\left(\frac{r_{2i}}{r_{2i}-a}\right)^2 - 1}$ $\frac{\lambda_i}{\lambda_0} = \left(\frac{b-a}{b}\right)$ | $\frac{\rho_{2i-1}}{\rho_0} = \frac{b}{b-a} \left(1 - \sqrt{1 - \left(\frac{r_{2i-1}-a}{r_{2i-1}}\right)^2}\right)$ $\frac{\rho_{2i}}{\rho_0} = \frac{b}{b-a} \left(1 + \sqrt{1 - \left(\frac{r_{2i}-a}{r_{2i}}\right)^2}\right)$ $\frac{\lambda_i}{\lambda_0} = \left(\frac{b-a}{b}\right)^2 \frac{r_i}{r_i-a}$ |
|                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                  |

**Figure 2:** Sound pressure field. Scatterer and fluid cloaking device under plane wave incidence.

This unfortunate characteristic of elastic media increases the difficulty to create a solid cloak since it implies that coordinate transformations cannot be implemented precisely as distribution of elastic properties. Despite this limitation, in 2010 Urzhumov et al. [8] presented a solid cloaking and studied the influence of the shear modulus. Urzhumov concluded that it was possible to implement a solid cloak under some restrictions. Moreover a bi-layer metamaterial model is introduced to facilitate the implementation of a solid cloak. Although this model does not require an anisotropic material it imposes several new constrains like thickness and filling fraction of the layers.

| Urzhumov's cloak                                                                                                                                                                           |  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| $\frac{\rho_r}{\rho_0} = \frac{r}{r-a}, \quad \frac{\rho_\phi}{\rho_0} = \frac{r-a}{r}$ $\frac{\lambda_l}{\lambda_0} = \left(\frac{b-a}{b}\right)^2 \frac{r}{r-a}$ $G = \lambda_l(1-2\nu)$ |  |
| <p>where <math>G</math> is the shear modulus, <math>\lambda_l</math> is the first Lamé coefficient and <math>\nu</math> is the Poisson's ratio.</p>                                        |  |

**Figure 3:** Sound pressure field. Scatterer and solid cloaking device under plane wave incidence.

## References

- [1] J. Pendry, D. Schurig and D. Smith: Controlling electromagnetic fields. *Science*, 2006. DOI: 10.1126/science.1125907.
- [2] S. Cummer, B. Popa, D. Schurig, D. Smith, J. Pendry: Full-wave simulations of electromagnetic cloaking structures. *Physical Review E*, 2006. DOI: 10.1103/PhysRevE.74.036621.
- [3] H. Chen: Transformation optics in orthogonal coordinates. *Journal of Optics A*, 2009. DOI:10.1088/1464-4258/11/7/075102
- [4] M. Yan, W. Yan, M. Qiu: Invisibility Cloaking by Coordinate Transformation. *Progress in Optics*, Elsevier, 2009. DOI: 10.1016/S0079-6638(08)00006-1.
- [5] S. Cummer and D. Schurig: One path to acoustic cloaking. *New Journal of Physics*, 2007. DOI: 10.1088/1367-2630/9/3/045.
- [6] Y. Cheng, F. Yang, J. Xu and X. Liu: A multilayer structured acoustic cloak with homogeneous isotropic materials. *Applied Physics Letters*, 2008. DOI: 10.1063/1.2903500.
- [7] H. Chen, T. Yang, X. Luo and H. Ma: Impedance-Matched Reduced Acoustic Cloaking with Realizable Mass and Its Layered Design. *Chinese Physics Letters*, 2008. DOI: 10.1088/0256-307X/25/10/049.
- [8] Y. Urzhumov, F. Ghezzo, J. Hunt and D. Smith: Acoustic cloaking transformations from attainable material properties. *New Journal of Physics*, 2010. DOI: 10.1088/1367-2630/12/7/073014.

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