Numerical simulation of transmission loss test facilities

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Numerical simulations for estimating the transmission loss can be an important alternative to measurements when there is no access to transmission loss test facilities. With such a tool, the properties of partitions can be varied without having to construct them, so that only the optimized variant would be built and measured. Due to the dimensions of a real test facility and the power of actual computers, conventional BEM or FEM can cover only the low and middle frequency range. The present work handles the source room, the partition and the receiver room separately. Starting from an initial state, the sound pressure in both rooms and the displacement of the partition are successively improved through the combination of separate calculations of sound radiation and structure vibration until a sufficiently accurate value is achieved. With this approach, instead of solving one large system, three smaller systems will be solved and higher frequencies can be handled. A significant gain can be obtained if the number of iterations remains low. This should be the case at frequencies apart from the resonance frequencies of both rooms and of the partition or at the resonance frequencies if a small damping is inserted in each system.

1 INTRODUCTION

Product developers are relying more and more on simulations at the design stage since the use of computational tools can lead to significant costs reduction. For the case of transmission loss measurements where big effort has to be spent to effectively close all openings connecting source and receiver rooms, numerical simulations represent an interesting alternative. Such simulations allow design optimization by means of “virtual measurements”, so that the best prototypes or the ones with the most interesting properties are selected for the real measurements. The calculations can be made using deterministic methods like Finite Element (FEM) and Boundary Element (BEM). Since the dimensions of real test facilities are not small,
the frequency range of the simulations is usually limited to low and middle frequencies. The extension of the range of application of these methods is a very important task. In the present work, an iterative method is developed which treats source room, partition and receiver room separately in order to reduce the size of the analyzed systems and extend the frequency range of application.

2 CALCULATION OF THE TRANSMISSION LOSS

The transmission loss ($R$) of the partition is calculated following the procedure described in DIN EN ISO 10140-2. The partition is placed between source and receiving room (see Fig. 1) and $R$ is defined as

$$R = L_1 - L_2 + 10 \log_{10} \left( \frac{S}{A} \right)$$

(Eqn. (2) in Ref. 1), where $L_1$ is the mean sound pressure in source room in dB (average of pressure in field points 1), $L_2$ mean sound pressure in receiving room in dB (average of pressure in field points 2), $S$ the area of the opening where the element is mounted (in m$^2$) and $A$ the equivalent absorbing area in the receiving room (in m$^2$).

![Fig. 1 - Illustration of the test facility.](image)

Equation (1) requires that the sound fields are diffuse and that the sound in the receiving room is exclusively due to the sound coming through the test element.

3 MOTION OF THE PARTITION

In this study we consider thin plates, i.e. the displacement is the same at both sides of the plate. For acoustic calculations, only the normal displacement of the plate $u_n$ is relevant. Since the motion of the plate is treated separately, different methods can be used to compute the normal displacement.

If a FEM calculation is performed, the equation of motion of the plate is given by

$$(K - \omega^2 M) \mathbf{u} = \mathbf{F}$$

where $K$ is the stiffness matrix, $M$ the mass matrix, $F$ the external force vector and $\mathbf{u}$ represents the vector of nodal displacements. The FEM mesh does not need to match the acoustic mesh, so that in general, the normal displacement (acoustic) can be defined as $u_n = Lu$, where $L$ is a matrix relating the nodal FEM displacements to the normal acoustic displacements.
For rectangular plates, a Ritz-Rayleigh approach can be applied. The normal displacement of the plate is expanded in a set of functions $\psi_{\mu \nu}^m$n$y$

\[ u_n = \sum_{\mu \nu} a_{\mu \nu} \psi_{\mu \nu} \]  

and the equation of motion of the plate is obtained by minimizing a functional. The minimization leads to the equation:

\[ (K - \omega^2 M) a = P \]  

where $a$ is the vector of unknown coefficients $a_{\mu \nu}$ and $P$ represents the generalized pressure acting on the plate. The normal displacement can be obtained at the acoustic mesh nodes using Eqn. (3) which in matrix form can be written as

\[ u_n = \Phi a \]  

where $\Phi$ is the matrix of the functions $\psi_{\mu \nu}$. In the following calculations, this approach was used to obtain the displacements of the plate.

### 4 SOUND FIELD IN THE ROOMS

The sound pressure in the rooms is calculated using the BEM. The whole surface of the acoustic domain is subdivided in 4 surfaces, $S_1$-$S_4$ with normal vectors $n_1$-$n_4$ as shown in Fig. 2. $S_2$ correspond to the plate and $S_3$ to the rigid surface between the rooms. On $S_1$ and on $S_4$ the impedances $Z_1$ and $Z_4$ are prescribed.

![Fig. 2 – BEM model for the test facility and the plate.](image)

The integral equations for the rooms are given by

\[ C_1 p_I = \int_{S_1} \left( p_{I1} \frac{\partial g}{\partial n_1} - j \rho \omega g \right) dS - \int_{S_2} \left( p_{I2} \frac{\partial g}{\partial n_2} - \rho \omega^2 g u_e \right) dS - \int_{S_3} p_{I3} \frac{\partial g}{\partial n_3} dS + p_q \]  

\[ C_3 p_u = \int_{S_1} \left( p_{u1} \frac{\partial g}{\partial n_1} - \rho \omega^2 g u_e \right) dS + \int_{S_2} \left( p_{u2} \frac{\partial g}{\partial n_2} \right) dS + \int_{S_3} p_{u3} \frac{\partial g}{\partial n_3} dS + \int_{S_4} \left( \frac{\partial g}{\partial n_4} - j \rho \omega g \right) dS \]

where $\rho$ is the density of air, $g$ the point source Green’s function, $p_{I1}$-$p_{I3}$ and $p_{u1}$-$p_{u3}$ the surface pressures and $p_q$ the sound pressure due to the sound source $q$.

Discretization of Eqs. (6) and (7) on the boundary leads to two matrix equations.
\[ A_i p^s_i = G_i \rho \omega^2 u_n + p_s \quad (8) \]
\[ A_\| p^s_\| = -G_\| \rho \omega^2 u_n \quad (9) \]

that provide the values of the sound pressure on the whole surface assuming \( u_n \) is known. Here \( p^s = [p_{11} \ p_{12} \ p_{13}]^T \) and \( p^s_\| = [p_{\|12} \ p_{\|13} \ p_{\|14}]^T \).

5 ITERATION SCHEME

Combining Eqns. (8) and (9) and replacing Eqn. (5) in Eqn. (3), an iterative scheme with a double step can be obtained:

\[ \Delta p^{(i)}_M = (L_f^I + L_f^\|)u^{(i-1)}_n \quad , \quad u^{(i)}_n = L_p(\Delta p^{(i)}_M + \Delta p_E) \quad (10) \]

In Eq. (10), \( \Delta p_M \) is the pressure difference due to the motion of the plate and \( \Delta p_E \) is the pressure difference due to the sound sources. \( L_p \) is the operator describing the motion of the plate and \( L_f^I \) and \( L_f^\| \) are the operators regarding the excitation of the plate due to the sound pressure.

The iteration starts assuming that the plate does not vibrate \( u^{(0)}_n = 0 \) (blocked-pressure approximation). Inserting this value in Eqn. (10), one obtains \( \Delta p^{(1)}_M = 0 \) and \( u^{(1)}_n = L_p \Delta p_E \). The calculations are then repeated until the difference between step \( i-1 \) and \( i \) is smaller than a certain value or the maximum number of iterations is reached.

The iterative approach will be useful if the solution converges to the right solution with a small number of iterations. Introducing some damping to the plate as well as absorption at the walls of both rooms, the number of iterations should decrease.

A combination of Eqs. (10) provides a single step iteration of the form

\[ u^{(i)}_n = T u^{(i-1)}_n + \chi \quad (11) \]

The iteration will converge if the spectral radius of the system matrix \( T \) is less than 1. The spectral radius is defined as \( \rho_m = \max(|\lambda_i|) \), where \( \lambda_i \) are the eigenvalues of \( T \).

6 NUMERICAL RESULTS

The window test facility of the “Institut für Bauphysik – Fraunhofer Institut” in Stuttgart, Germany was considered. The source and receiving rooms are rectangular rooms with dimensions 5.74m \( \times \) 3.75m \( \times \) 3.11m and 4.85m \( \times \) 3.75m \( \times \) 3.11m respectively. The opening is also rectangular with dimensions 1.25m \( \times \) 1.5m. For simplicity, the width of the wall dividing the two rooms was neglected. In the source room a small absorption \( \alpha = 0.18 \) was considered while in the receiving room a high absorption \( \alpha = 0.89 \) was used.

A point source was placed near one corner of the source room. Field points 1 are placed on a sphere of radius 0.75m centered approximately in the middle of the source room. Field points 2 were set 0.5m away from the plate (see Fig. 1).

6.1 Comparison between Direct and Iterative Methods

For verification of the iterative approach, the TL of an aluminum plate obtained with the iterative approach was compared with the TL obtained by solving the coupled problem (direct approach). The simulated plate has the dimensions of the opening and a thickness of 0.004m.
aluminum, the values taken for Young’s modulus, density and Poisson’s ratio are: $E = 64 \cdot 10^9$ Pa, $\rho_p = 2,700$ kg/m$^3$ and $\nu = 0.3$. Damping was considered in the plate ($\eta = 0.05$).

**Fig. 3 – Comparison between direct and iterative methods.**

Figure 3 shows the results of the numerical simulation. The transmission loss obtained with the iterative approach is illustrated together with the results of a direct calculation on the top left plot. Both curves are practically identical, because the differences are very small, below 0.3 dB as shown in the bottom left plot. On the right side, one can see on the top the spectral radius of the system matrix $T$ of Eqn. (11) and on the bottom the number of iterations needed. Since the spectral radius is smaller than 1 for all frequencies, convergence of the iterative method is ensured. The number of iterations is higher for values of the spectral radius near 1 than for lower values as expected.

**Fig. 4 – Pressure level distribution.**
Figure 4 shows the sound pressure level in both rooms for four different frequencies, 60 Hz, 80 Hz, 100 Hz and 150 Hz. It is easy to recognize the presence of the resonances in the source room, since the absorption is small. The resonances are also present in the receiver room but they are less pronounced due to the higher absorption of the walls. The condition of a diffuse field in both rooms for the validity of Eqn. (1) is not satisfied.

6.2 Sensitivity to the Wall Absorption

To attenuate the resonances in the source room, the absorption was increased, while the absorption in the receiver room remained the same. Two additional absorption coefficients $\alpha = 0.64$ and $\alpha = 0.89$ were tested. The pressure level distribution is presented in Fig. 5. The higher absorption helps a bit but still no good diffusivity is obtained.

6.3 Sensitivity to the Thickness of the Plate

The thickness of the plate was varied to see the influence on the spectral radius and on the convergence of the iterative method. All other parameters of the calculation did not change. Two
additional values of the thickness were studied, \( h = 0.002 \text{ m} \) and \( h = 0.001 \text{ m} \). In Fig. 6, the curves of the spectral radius for the three cases are compared. The three curves show the same behaviour, i.e. large oscillations at low frequencies and smaller oscillations together with a uniform decay at high frequencies. The curves are shifted up by decreasing thickness.

![Fig. 6 – Spectral radii.](image)

In Fig. 7, the curves of transmission loss for the two additional cases are shown as well as the corresponding spectral radii. At the frequencies where \( \rho_m > 1 \), the deviations with respect to the results of the direct calculation are usually very large and for the other frequencies good agreement can be observed.

![Fig. 7 – Transmission loss and spectral radii.](image)
7 CONCLUSIONS

This paper presents an iterative approach to simulate transmission loss measurements. The method solves three separate smaller systems instead of one large system. When the spectral radius of the system matrix is less than 1, the method converges in only a few iterations. The numerical tests show that convergence problems are present for very thin structures.

The condition of diffuse sound fields is not satisfied, especially if the test facility is made of rectangular rooms. Increasing the absorption of the walls helps slightly. Ways to increase the diffusivity and to ensure the convergence need to be investigated.

The calculations were performed up to 800 Hz with a model of 24000 elements. To cover a higher frequency range, larger models are needed. For such big models, instead of using the direct BEM for the sound pressure calculation, the Fast Multipole BEM can be easily inserted in the method. This task will be performed in a future work.

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9 REFERENCES

