EURONOISE Prague 2012

Calculation of the transmission loss of thin plates in a Kundt's tube

Rafael Piscoya, Martin Ochmann
Beuth Hochschule für Technik Berlin, University of Applied Sciences, Berlin, Germany.

Summary
The transmission loss of a thin plate in a duct is computed by a coupled Rayleigh-Ritz-BEM approach. The normal vibrations of the plate are calculated with a Rayleigh-Ritz approach. They are used as basis for the modal expansion of the displacement of the plate. The participation factors are determined solving the equation of motion of the plate including the fluid load. The load acting on the plate due to the sound pressure is determined using the BEM. The results of the model are compared to the results obtained by an approximated approach which treats the plate as a piston whose velocity is the mean velocity of the plate. The simulated transmission loss is compared with a measurement for validation. Two methods found in the literature to determine the transmission loss in a duct which has reflecting terminations are also investigated.

PACS no. 43.55.Ti, 43.20.Rz

1. Introduction

The transmission loss (TL) of thin plates is usually determined by measurements in TL test facilities which consist of two adjacent rooms connected through an opening where the specimen is placed. These rooms require large spaces and are expensive. When there is no access to such facility, an alternative way to obtain the TL is to measure smaller samples inside a Kundt's tube. The frequency range of validity of the measurements lies below the first cut-on frequency of the tube, since propagation of plane waves is assumed. For small samples, the form of fixation to the walls of the tube (boundary conditions) has an important influence on the vibration of the plate and hence on its transmission coefficient. In this work, the sound field inside the Kundt's tube is simulated using the Boundary Element Method and the vibration of the plate is determined using a Rayleigh-Ritz method which allows the inclusion of elastic boundary conditions. The method considers full acoustic structure interaction. Simply supported, clamped plates and plates fixed to the tube with springs will be studied. In the latter case, the elastic constants are chosen so that the results of the simulation agree with the corresponding measurements.

2. Equation of motion of the plate

We consider a rectangular plate of sides a and b and thickness h which is supported by translational and rotational springs with translational and rotational rigidities K_b and C_b respectively (see Fig. 1).

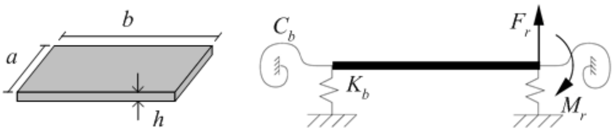

Figure 1. Plate dimensions and boundary support.

Arbitrary BCs can be studied by setting different values to the rigidities. The usual BCs are defined by the values K_b=0 and C_b=0 (free), K_b>>1 and C_b=0 (simply supported) and K_b>>1 and C_b>>1 (clamped).

The displacement of the plate u is calculated with a variational principle. Within this approach, the plate vibrations correspond to minimum values of the action functional H [1]:

$$H = \int_{t_0}^{t_1} (T - V - V_b)\,dt + \int_{t_0}^{t_1} W dt \quad , \tag{1}$$

where T is the kinetic energy, V the deformation energy of the plate, V_b the potential energy on the boundary and W is the work of the external forces.

If u is developed into a modal or polynomial basis f_{nm} and the infinite series is truncated after N terms

$$u = \sum_{n,m=1}^{N} a_{nm} f_{nm} \;,\quad f_{nm}(x,y) = j_n(x) j_m(y)\,, \tag{2}$$

an equation for the a_{nm} can be obtained:

$$(K_{pqnm} - w^2 M_{pqnm}) a_{nm} = F_{pq} \qquad (3)$$

where K_{pqnm} is the stiffness matrix, M_{pqnm} the mass matrix and F_{pq} the vector of external forces.

3. Sound field in the tube

Below the first „cut-on-frequency“ (f_{c1}), only plane waves propagate since the modes of higher order are strong attenuated (Fig. 2). The latter produce the acoustic near field of the plate. For a duct with square cross section of side b, the first cut-on-frequency is f_{c1}=c/2b.

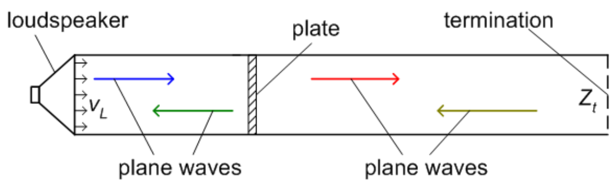

Figure 2. Sound field in Kundt's tube.

When a partition with known impedance Z_{part} is placed in an infinite duct, a simple expression for the transmission coefficient t can be deduced [2]:

$$t = \left| \frac{2Z_0}{Z_{part} + 2Z_0} \right|^2 \quad , \quad Z_0 = rc \qquad (4)$$

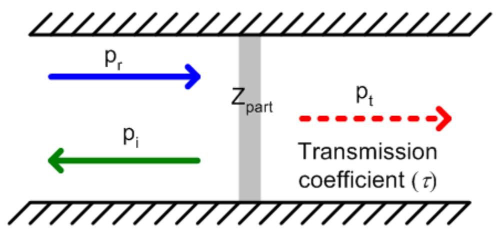

Figure 3. Partition in an infinite duct.

4. Approximation: plate as a piston

Below f_{c1}, the net effect of the plate vibration will be similar to the effect of a piston. Under this assumption, (4) can be used to predict the transmission coefficient of the plate by replacing Z_{part} with the mean impedance of the plate Z_P [3]. The displacement of the plate u_P due to a uniform pressure Δp can be obtained solving (3) with the right side F_{pq}

$$F_{pq} = \int_S f_{pq} \Delta p dS \quad . \qquad (5)$$

The mean impedance Z_P is defined as

$$Z_P = \frac{\Delta p}{\langle \dot{u}_P \rangle} \text{ , with } \langle \dot{u}_P \rangle = \frac{1}{S} \int_S \dot{u}_P dS \quad , \qquad (6)$$

5. Numeric model

A numeric procedure to compute the sound field inside a duct based on the Boundary Element method (BEM) was implemented. It is not limited to frequencies below f_{c1}. The limitation is given by the discretization of the surface ($\Delta x \leq \lambda/6$).

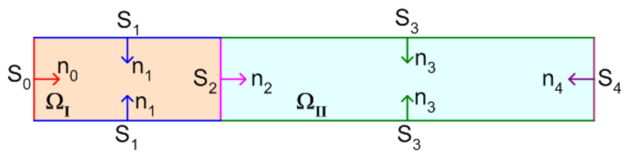

Figure 4. Sound field in Kundt's tube.

Fig. 4 shows the two computational domains Ω_{I} and Ω_{II} and the five different surfaces of the model with their respective normal vectors. The BCs on all surfaces are listed in Table 1.

Table I: Boundary conditions

Surface	Description	Boundary condition
S_0	loudspeaker	v_n=v_L
S_1, S_3	side wall	v_n=0
S_2	plate	$\partial p_I/\partial n = \partial p_{II}/\partial n = r w^2 u_P$
S_4	termination	Z=Z_t

The boundary integral equations in Ω_{I} and Ω_{II} are given by

$$C_I p_I = \int_{S_0} \left(p_{I0} \frac{\partial g}{\partial n} + jrwv_L g \right) dS + \int_{S_1} p_{I1} \frac{\partial g}{\partial n} dS - \int_{S_2} \left(p_{I2} \frac{\partial g}{\partial n} - rw^2 u_P g \right) dS \qquad (7)$$

$$C_{II} p_{II} = \int_{S_2} \left(p_{II2} \frac{\partial g}{\partial n} - rw^2 u_P g \right) dS + \int_{S_3} p_{II3} \frac{\partial g}{\partial n} dS + \int_{S_4} p_{II2} \left(\frac{\partial g}{\partial n} - \frac{jrwg}{Z_t} \right) dS \qquad (8)$$

with $C = \begin{cases} 1 & \text{inside } \Omega \\ 0.5 & \text{on S} \\ 0 & \text{outside } \Omega \end{cases}$ and $g = \dfrac{e^{-jk|\vec{x} - \vec{y}|}}{4p|\vec{x} - \vec{y}|}$.

The excitation of the plate is proportional to the pressure difference

$$F_{pq} = \int_S f_{pq}(p_{I2} - p_{II2})dS \quad . \qquad (9)$$

Discretizing (7) and (8) on all surfaces and introducing the expansion (2) for u_P, p_{I2} and p_{II2} can be expressed in terms of the coefficients a_{nm} and the two terms on the right hand side in (9) can be written as

$$\begin{aligned} \int_S f_{pq} p_{I2} dS &= -jwZ_{Ipqnm} a_{nm} + P_{pq} \\ \int_S f_{pq} p_{II2} dS &= jwZ_{IIpqnm} a_{nm} \end{aligned} \qquad (10)$$

By combining (3), (9) and (10), we obtain the system of equations for the coefficients a_{nm}:

$$(K_{pqnm} - w^2 M_{pqnm} + jwZ_{Ipqnm} + jwZ_{IIpqnm}) a_{nm} = P_{pq} \; . \qquad (11)$$

6. Transmission loss of the plate

The transmission loss (TL) of the plate is defined as

$$TL = -10\log_{10}(t) \; , \quad t = \frac{W_t}{W_i} \quad , \qquad (12)$$

where t is the transmission coefficient and W_t and W_i are the transmitted and incident sound power respectively. In the numeric model, the TL is obtained emulating the measurement procedure. Only plane wave propagation is assumed, therefore this method is valid only below f_{c1}. For higher frequencies, another definition of TL is needed.

We consider two different configurations: 1) with anechoic termination and 2) with reflecting termination

6.1. Anechoic termination

In this case, in the downstream section of the duct only a plane wave in the +x direction propagates while in the upstream section two plane waves propagate in the +x and –x directions respectively (see Fig. 5).

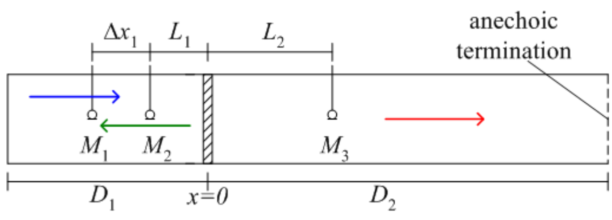

Figure 5. Microphone positions by a non-reflecting termination.

If A, B and C are the amplitudes of the plane waves, the transmission coefficient of the plane waves will be $t = |C|^2/|A|^2$. t can be expressed in terms of the sound pressure at the microphones p_{M1}, p_{M2} and p_{M3} as:

$$t = \frac{|2\sin k\Delta x_1|^2 |p_{M_3}|^2}{|p_{M_1} - p_{M_2} e^{-jk\Delta x_1}|^2} \qquad (13)$$

The results of (13) will be accurate provided $\Delta x_1 < c / f$.

6.2. Reflecting termination

An anechoic termination is in practice difficult to achieve. Fortunately, also in that case, the TL can be determined. When the termination reflects sound, there is also a plane wave propagating in the –x direction in the downstream section of the duct and it must be taken into account. Therefore, an additional microphone has to be used (see Fig. 6).

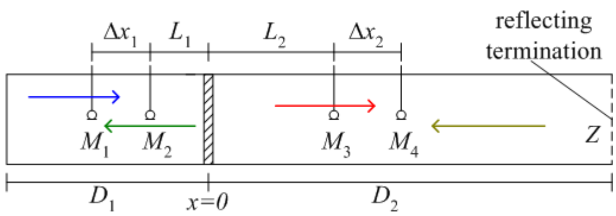

Figure 6. Microphone positions by a reflecting termination.

In the literature, one can find two types of methods to determine the TL of arbitrary acoustic elements, one based on the transfer matrix [4] and the other based on the wave decomposition [5].

6.2.1 Transfer matrix (TM)

The amplitudes of the plane waves A, B, C and D can be related in a matrix formulation as

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} a & b \\ g & d \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} \quad . \qquad (14)$$

The coefficients of the matrix a, b, g and d depend only on the physical properties of the acoustic element. If the termination is anechoic (D=0), then a=A/C and t=|1/a|^2. Therefore, for the sound transmission, the relevant coefficient is a .

The four coefficients require four equations. Eq. (14) represents two equations. The other two equations can be obtained by changing the termination of the duct. Since two different terminations are used, this method is regarded as a "*two-load technique*".

If we denote with I and II the different configurations, we can deduce a new system of equations

$$\begin{pmatrix} A^I \\ A^{II} \end{pmatrix} = \begin{pmatrix} C^I & D^I \\ C^{II} & D^{II} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad , \tag{15}$$

and using the determinant method, the coefficient a can be obtained

$$a = \frac{A^I D^{II} - A^{II} D^I}{C^I D^{II} - C^{II} D^I} \tag{16}$$

6.2.2 Wave decomposition (WD)

In the upstream section of the duct, the sound pressure is given by

$$p_u = p_u^+ + p_u^- = A(e^{-jkx} + r_1 e^{jkx}) \quad , \tag{17}$$

where r_1 is the reflection coefficient of the element plus the downstream section of the duct.

The total incident sound pressure on the front side of the plate is A, hence the sound pressure transmitted to the downstream section of the tube will be At, where t is the "complex" transmission coefficient ($t = |t|^2$). The total wave propagating in the $+x$ direction can be written as

$$p_d^+ = Ate^{-jkx}(1 + r_2 r_b e^{j2kD_2} + (r_2 r_b e^{j2kD_2})^2 + \ldots)$$
$$= \frac{Ate^{-jkx}}{1 - r_2 r_b e^{j2kD_2}} \quad , \tag{18}$$

where r_2 is the reflection coefficient due to the element plus the upstream section of the duct and r_b is the reflection coefficient of the termination. Similarly, the total wave propagating in the $-x$ direction can be written as

$$p_d^- = \frac{Atr_b e^{jkx}}{1 - r_2 r_b e^{j2kD_2}} \quad . \tag{19}$$

Eqs. (17)-(19) contain five unknown coefficients A, r_1, t, r_2, r_b. With the four microphone technique, only four independent equations are obtained. By changing the termination of the duct, there will be four additional equations and three new coefficients A', r'_1 and r'_b (t and r_2 remain the same). Hence, in a similar way as in the TM method, two measurements are needed to calculate the transmission coefficient (two-load technique). t is written as:

$$t = \left| \frac{p_{M_3}}{p_{M_2}} (1 - r_2 r_b e^{j2kD_2}) \frac{(e^{jkL_1} + r_1 e^{-jkL_1})}{(e^{-jkL_2} + r_b e^{jkL_2})} \right|^2 \tag{20}$$

7. Numerical example

The transmission loss of a 1 mm thick aluminium plate was simulated. The plate was placed in a 4 m long duct with square cross section (0.25 m side length).

In a first calculation, a duct with an anechoic termination was considered. In that case, an impedance $Z=\rho c$ on S_4 was assumed. The transmission coefficient was computed using (13) and the sound pressure in the duct was determined using (7) and (8). To validate the numerical model, the results of the simulation were compared with the results of a measurement. Since the plate was fixed to the duct walls using an adhesive material, an "elastic BC" was assumed. The rigidities of the translational and rotational springs were chosen in such a way that a good agreement between simulation and measurement is obtained.

Fig. 7 shows the comparison of simulated and measured TLs. The approximated TL obtained with (4) is almost identical to the TL computed with (13) from the numerical model. The biggest difference occurs at low frequencies, below 50 Hz.

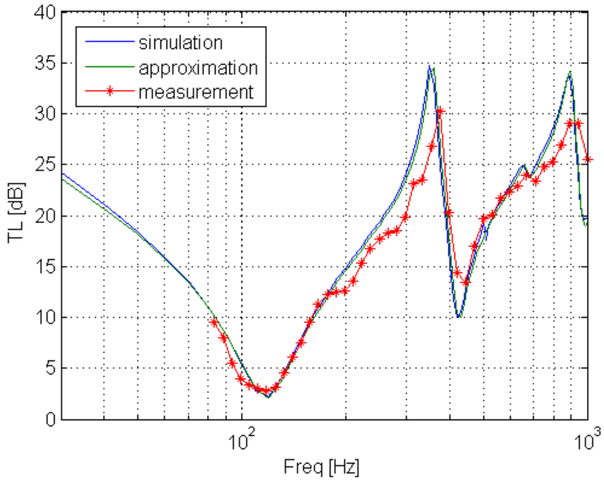

Figure 7. Comparison of simulation and measurement.

In Fig. 8, the TL of the plate for different BCs is illustrated. It can be observed, that the BCs exert a strong influence on the TL.

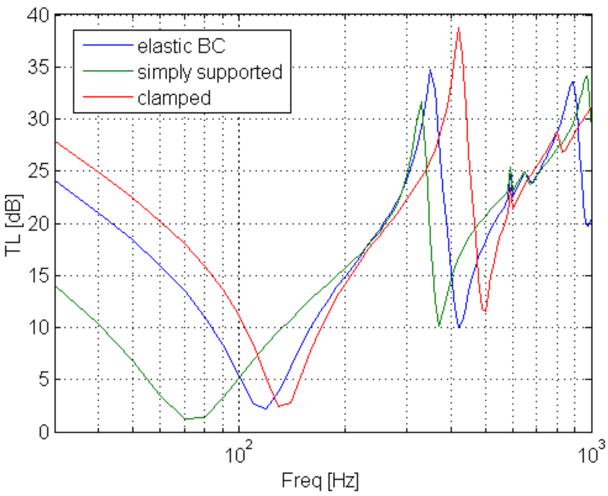

Figure 8. TL for different BCs.

The displacement of the plate for three different frequencies is depicted in Fig. 9. The pattern of deformation is complicated and can excite higher modes in the duct near the plate, but since they are strongly attenuated, they disappear at a short distance.

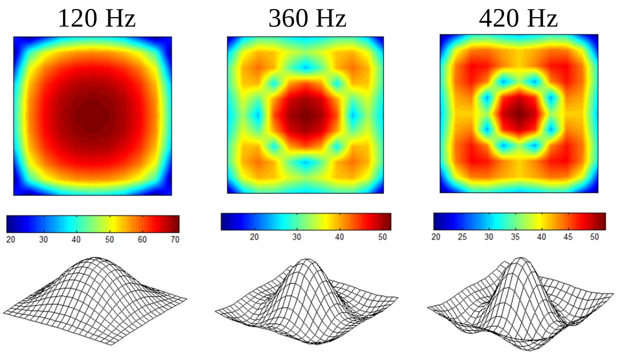

Figure 9. Displacement of the plate.

In a second calculation, a duct with a reflecting termination was considered. As described in section 6, two different impedances are required. We use the impedances

$$Z_I/rc = 0.3(1+j) \quad \text{and} \quad Z_{II}/rc = +0.1(1-j)\ .$$

The TL was calculated applying the TM and WD approaches and the results were compared.

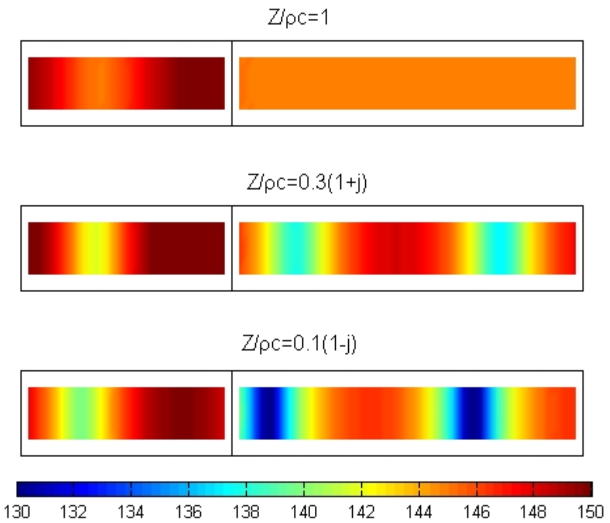

Figure 10. Pressure level in the duct for different terminations.

In Fig. 10, the pressure distribution inside the duct at 120 Hz for three different terminations is shown. For an anechoic termination ($Z=rc$), there is only a plane wave travelling in the $+x$ direction and the pressure level is constant. For a reflecting termination, waves travelling in both directions exist and zones of maxima and minima can be observed.

Fig. 11 compares the TL of the same plate obtained with different terminations. The TM and WD approaches for reflecting termination provide identical results and they differ minimally from the result obtained with the anechoic termination.

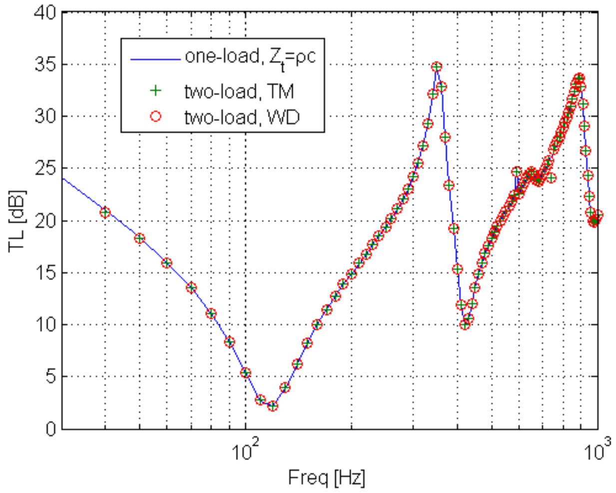

Figure 11. Simulated TL for the one- and two-load technique.

8. Conclusions

A numerical model to determine the transmission loss of a plate in a duct based on a coupled Rayleigh-Ritz-BEM method was implemented. The approach accepts arbitrary boundary conditions for the plate and emulates measurements with three or four microphones depending on the termination of the duct. By means of this method it was possible to evaluate the accuracy of the approximation that considers the plate as a piston. The results indicate that this assumption is valid. The numerical model provides results that agree well with the measurements but it was found that a correct estimation of the boundary condition of the plate is very important. Finally, it was shown that it is not necessary to have an anechoic termination in the duct to determine accurately the transmission loss of an element. However, two different terminations and two separate measurements are required. The TL can be calculated with either the TM or the WD approach since both are equivalent.

Acknowledgements

This work was supported by the German Research Foundation (DFG) within the project "General computational model for sound transmission problems".

The authors would like to thank Mrs. Aoki from the Institut für Bauphysik in Stuttgart for providing the measurement data.

Literature

[1] R. Woodcock, J. Nicolas: A generalized model for predicting the sound transmission properties of generally orthotropic plates with arbitrary boundary conditions. J. Acoust. Soc. Am. 97(2) (1995), 1099-1112.

[2] X. Zhou, H. Drotleff, M. Leistner, T. Zhang: Bestimmung des Dissipations- und Transmissionsgrades dünner, endlicher Schichten bei senkrechtem Schalleinfall. Fortschritte der Akustik, DAGA 2007.

[3] Y. Aoki: Sound Transmission through a thin Aluminium panel in a Kundt's Tube. Intern report, 2009.

[4] P. Bonfiglio, F. Pompoli: A single measurement approach for the determination of the normal incidence transmission loss. J. Acoust. Soc. Am. 124(3) (2008), 1577-1583.

[5] Y. Salissou, R. Panneton: A general wave decomposition formula for the measurement of normal incidence sound transmission loss in impedance tube. J. Acoust. Soc. Am. 125(4) (2009), 2083-2090.