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SOUND PROPAGATION IN A REGION OF HOT GAS USING THE DRBEM

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Abstract

Simulations of the sound from flames inside combustion chambers define their computational domains predominantly inside the chamber, where the main acoustic sources and their interactions with the combustion processes take place. However, the noise pollution is determined by the sound radiated to the outside. Therefore, the prediction of the sound radiation, in particular of the radiation pattern outside the combustion chamber is of great importance, too. Immediately outside the chamber exit, there is a region of hot gas surrounded by homogeneous medium (air at ambient temperature). Knowing one acoustic quantity (pressure, velocity or impedance) at the exterior surface of the combustion chamber including the exit, the sound radiation can be obtained by solving a Helmholtz-Integral-Equation with an additional volume integral over the region of hot gas. Our research intends to study the applicability of the Dual Reciprocity BEM (DRBEM) to solve this problem.

In this work, the DRBEM is applied to find the sound radiation of a round flame, modelled as a spherical volume of hot gas with a certain source distribution surrounded by air at ambient temperature. If the sound distribution depends only on the distance to the origin, the 3D-problem can be reduced to a one dimensional problem and analytically solved for specific source distributions. A comparison of the results of the DRBEM with the analytical solution of this spherical configuration is presented here as a test case.

1. INTRODUCTION

A flame emitting sound can be modeled in a simple form by a spherical volume of hot gas with a certain sound source distribution, surrounded by air at room temperature [1]. If the sound source distribution depends only on the distance to the sphere center, the three-dimensional problem can be reduced to a one-dimensional problem and for specific source distributions, analytically solved. In the present work, the Dual Reciprocity Boundary Element Method (DRBEM) will be implemented in a numerical procedure in order to evaluate the volume integral over the source function with a low computational effort. The accuracy of the method is examined by comparing numerical and analytical results of a constant and two radial source distributions. The two not constant source distributions complement the results of a previous article [2]. In future works, the method will be applied to handle more complex geometries and arbitrary source distributions.

2. DESCRIPTION OF THE PROBLEM

The flame model consists of a spherical volume of hot gas with radius a, density ρ_1, sound speed c_1 and a (frequency dependent) sound source distribution Q_ω. The flame is surrounded by air with constants ρ_0 and c_0.

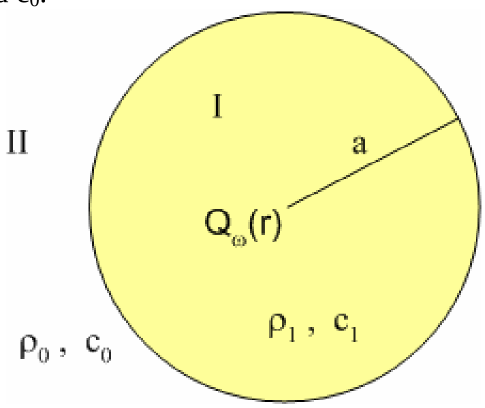

Figure 1: Spherical flame.

To determine the sound radiation, the space is divided in two regions and the Helmholtz equation is solved in each region [1]

$$\begin{aligned} \left(\nabla^2 + k_1^2\right)p_I &= Q_\omega \quad \text{Region I} \\ \left(\nabla^2 + k_0^2\right)p_{II} &= 0 \quad \text{Region II} \end{aligned} \tag{1}$$

with the boundary conditions

$$\begin{aligned} p_I &= p_{II} \quad , \quad r = a \\ \frac{1}{\rho_1}\frac{\partial p_I}{\partial r} &= \frac{1}{\rho_0}\frac{\partial p_{II}}{\partial r} \quad , \quad r = a \end{aligned} \tag{2}$$

3. ANALYTICAL AND NUMERICAL SOLUTIONS

3.1 Analytical solution

For special Q_ω, the problem can be solved analytically. We study first the case Q_ω= -Q, with Q constant for all frequencies. The solution for this case is:

$$\begin{aligned} p_I &= Q(Aj_0(k_1 r) - 1)/k_1^2 \quad r \le a \\ p_{II} &= Te^{-jk_0 r}/r \quad r \ge a \end{aligned} \tag{3}$$

where the constants A and T are determined by the boundary conditions (2).

3.2 Numerical solution

Using the usual boundary element procedure, the differential equations in (1) are transformed into their integral form:

$$\begin{aligned} C_I p_I &= -\int_S \left(p_I^S \frac{\partial g_1}{\partial r} - \frac{\partial p_I^S}{\partial r} g_1 \right) dS + \int_V Q g_1 dV \\ C_{II} p_{II} &= \int_S \left(p_{II}^S \frac{\partial g_0}{\partial r} - \frac{\partial p_{II}^S}{\partial r} g_0 \right) dS \end{aligned} \tag{4}$$

with

$$g_0 = \frac{e^{-jk_0 R}}{4\pi R} \quad , \quad g_1 = \frac{e^{-jk_1 R}}{4\pi R} \quad , \quad R = \left| \vec{x} - \vec{y} \right|$$

and

$$C_I = \begin{cases} 1 & r < a \\ 0.5 & r = a \\ 0 & r > a \end{cases} \quad , \quad C_{II} = \begin{cases} 0 & r < a \\ 0.5 & r = a \\ 1 & r > a \end{cases} \quad , \quad r = \left| \vec{x} \right|$$

By means of the DRBEM, the volume integral over the sound source distribution in Eq. (4) will be rewritten as a series of surface integrals to reduce the computational costs.

This is obtained by first expanding the source distribution in a series of functions f_j:

$$Q(\vec{x}) = \sum_{j=1}^{N+L} \alpha_j f_j(\vec{x}) \tag{5}$$

where N is the number of points at the spherical surface and L the number of points inside the volume (see Fig. 2). The basic idea of the method consists in finding some functions ψ_j that are solutions of an inhomogeneous Helmholtz equation with function f_j as source term:

$$\left(\nabla^2 + k^2 \right) \psi_j = f_j \quad . \tag{6}$$

A second application of the boundary element procedure leads to the following relation:

$$-\int_V f_j g dV = C \psi_j + \int_S \left(\psi_j^S \frac{\partial g}{\partial r} - \frac{\partial \psi_j^S}{\partial r} g \right) dS \, . \tag{7}$$

Replacing Eqs. (7) and (5) in (4), the integral equations in (3) can be written in terms of surface integrals alone:

$$C_I p_I = -\int_S \left(p_I^S \frac{\partial g_1}{\partial r} - \frac{\partial p_I^S}{\partial r} g_1 \right) dS + \sum_j \alpha_j \left(C \psi_j + \int_S \left(\psi_j \frac{\partial g_1}{\partial r} - \frac{\partial \psi_j}{\partial r} g_1 \right) dS \right) \tag{8}$$

More details of the method can be found in [3].

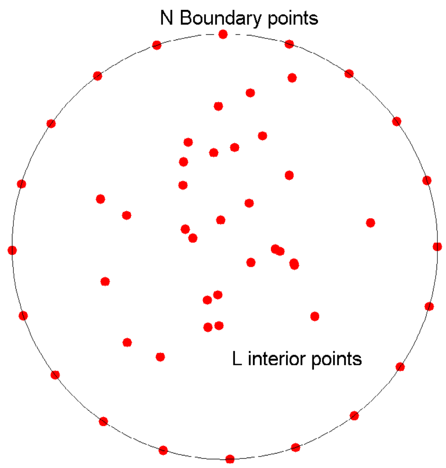

Figure 2: Discretization points of the DRBEM.

4. RESULTS

4.1 Constant source distribution

There are different functions f_j and ψ_j that can be selected, depending on the form of the source term Q_ω. In the present work, the pair proposed in [4] was used:

$$f_j(\vec{x}) = 1 + r_j \quad , \quad r_j = \left|\vec{x} - \vec{y}_j\right|$$
$$\psi_j(\vec{x}) = \frac{1+r_j}{k^2} - \frac{2}{k^4}\left(\frac{1-\cos(kr_j)}{r_j}\right) \tag{9}$$

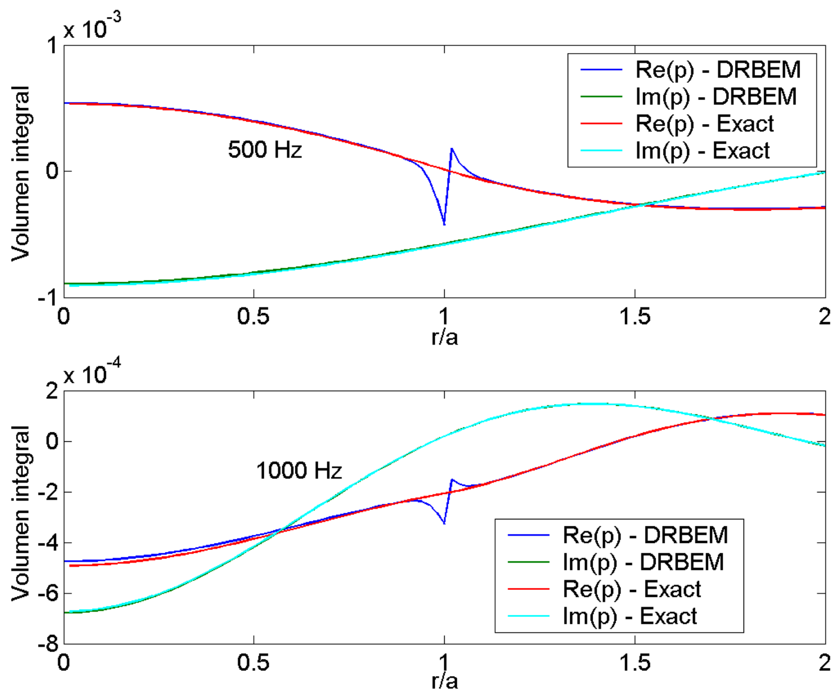

Figure 3: Volume integral computed at different positions.

The spherical surface had 640 elements and for the radius considered, there are at least six elements pro wavelength up to about 1000 Hz. However, due to the symmetry of the problem, good results are obtained for frequencies above this limit. Inside the sphere, 200 points where taken (N=640, L=200) which showed to be enough to reproduce the source distribution.

The agreement between analytical and numerical results is excellent. Fig. 3 shows a comparison of the analytical and theoretical values of the volume integral in different points inside and outside the sphere, while Fig. 4 illustrates a similar comparison of the sound pressure. Only at the interface between Region I and II, the error of the numerical calculations is noticeable, but this error does not affect the sound power (see Fig. 5).

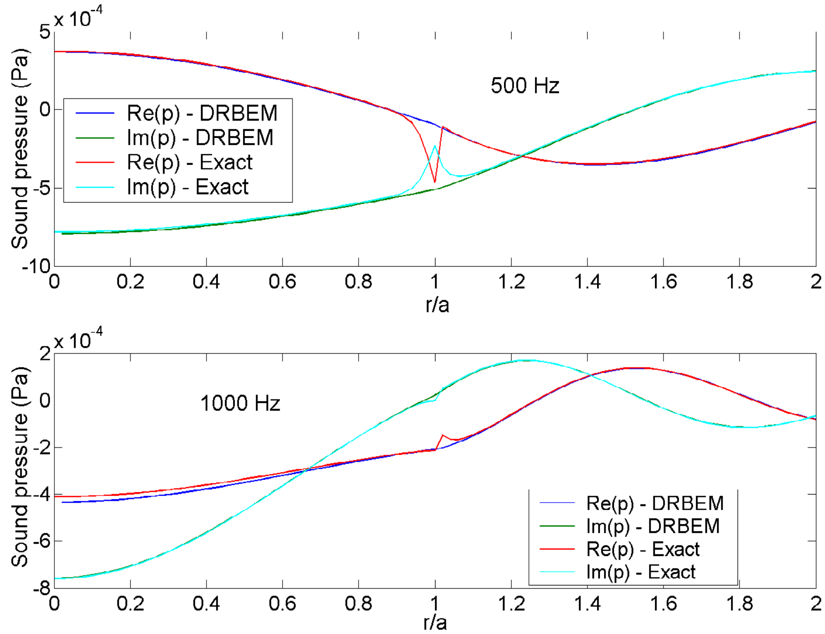

Figure 4: Sound pressure computed at different positions.

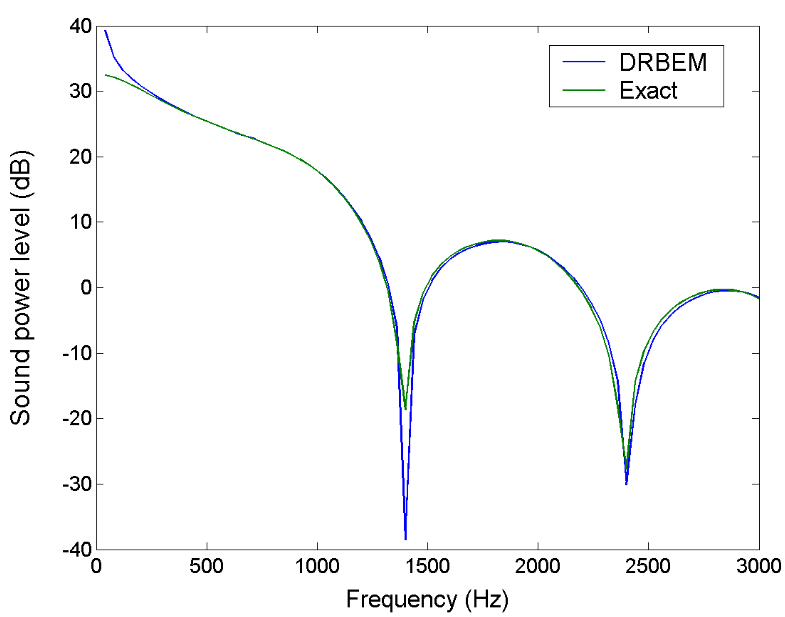

Figure 5: Sound power level in the frequency domain.

4.2 Not constant source distributions

Two other radial distributions, which have also analytical solution, were investigated. The solution in region II is again: $p_{II} = Te^{-jk_0 r}/r \quad r \geq a$ and the constants A and T are obtained by using the boundary conditions.

4.2.1 Exponential distribution:

Considering a distribution of the form: $Q_\omega = -Qe^{-br}$, the solution in region I is given by:

$$p_I = Q\left(Aj_0(k_1 r) + \frac{2b}{\left(k_1^2 + b^2\right)^2}\left(\frac{\cos(k_1 r) - e^{-br}}{r}\right) - \frac{e^{-br}}{k_1^2 + b^2}\right) \quad r \leq a$$

It is easy to see that for b=0, we recover the constant case.

4.2.2 Cosine distribution

Taking a distribution of the form: $Q_\omega = -Q(1 + \cos(k_s r))$, the sound pressure in region I is given by:

$$p_I = Q\left(Aj_0(k_1 r) - \frac{2k_s^2 j_0(k_s r)}{\left(k_1^2 - k_s^2\right)^2} - \frac{\cos(k_s r)}{k_1^2 - k_s^2} - \frac{1}{k_1^2}\right) \quad r \leq a$$

for $k_s \neq k_1$. One can note, that for k_s=0, we have the constant case with a source twice as loud as the original case.

In Fig. 6, the sound power for both not constant distributions is presented. For the calculations, the parameters b and k_s were set to 16.66 and 10.47 respectively.

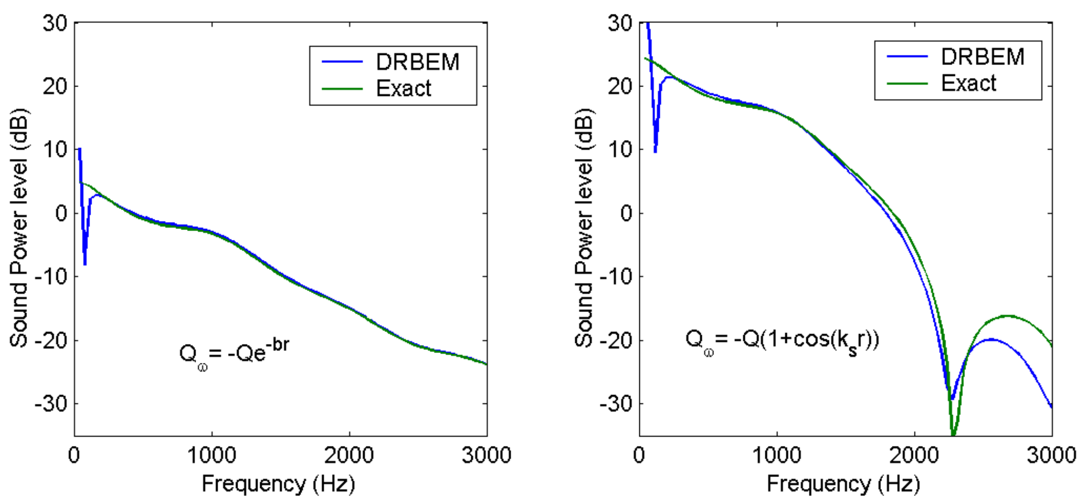

Figure 6: Sound power level in the frequency domain.

For this two source distributions, the agreement is once again very good. Here, as well as in the constant case, the error is noticeable at very low frequencies, but above 150 Hz, the method provides excellent results. For the cosine-distribution, the accuracy of the method decreases above 2200 Hz, but since for those frequencies the rule of six elements pro wavelength is not fulfilled, such error is expected.

5. AKNOWLEDGEMENTS

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6. SUMMARY

A numerical method based on the DRBEM have been implemented to calculate the sound radiation in regions where a known source distribution is present. The method was tested by using it to determine the sound radiation of a round noisy flame, which was represented as a volume of hot gas with a distributed sound source. The good agreement between numerical and theoretical results for three different source distributions validates the application of this approach and promotes its use in problems with more complex geometries. We intend to extend the utilization of integral methods to handle problems involving not only non reactive flows [3] but also reactive flows (flames) [5].

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