19th INTERNATIONAL CONGRESS ON ACOUSTICS
MADRID, 2-7 SEPTEMBER 2007

DETERMINATION OF THE SOUND RADIATION OF TURBULENT FLAMES USING AN INTEGRAL METHOD

PACS: 43.20.Rz

Piscoya, Rafael; Ochmann, Martin
University of Applied Sciences; Luxemburger Str. 10, 13353 Berlin, Germany;
piscoya@tfh-berlin.de , ochmann@tfh-berlin.de

ABSTRACT

An efficient way to determine the far field radiation of turbulent flames is by using hybrid approaches that couple full non linear flow equations solvers (CFD codes) with linear propagation acoustic methods. One possible acoustic method is the Lighthill's acoustic analogy which expresses the sound pressure in terms of a volume integral over the sound source distribution. This method has a high computational cost since the volume occupied by the sound sources has to be discretized. Purpose of this study is to develop a method that reduces the time of computation by rewriting the volume integral in terms of surface integrals alone. In this work, the basic ideas of the method are presented and the accuracy of the procedure is tested using a simple configuration that has an analytical solution.

INTRODUCTION

In previous works [1],[2], a hybrid method coupling an incompressible Large Eddy Simulation (LES) with the Equivalent Source Method (ESM) and the Boundary Element Method (BEM) has been used to determine the sound radiation of open turbulent flames. The velocity distribution over a cylindrical surface (control surface) surrounding the flame was obtained by the LES and transferred to the ESM and BEM as a Neumann boundary condition. From the spectrum of these unsteady data, the sound power and the radiation patterns of the flame were computed in a frequency range extending from 40 Hz to 5000 Hz. Conditions for the validity of the method are: 1) all sources should be enclosed by the control surface and 2) outside the cylindrical surface, the medium should be homogeneous. The first condition is easier to fulfil than the second one, particularly by jet flames where the region of non uniform mean velocity could extend downstream tens of times the nozzle diameter. In our case, the size of the LES computational domain was extended as long as possible trying to diminish the effect of the non uniformity of the medium and keeping the calculation time in reasonable limits (LES computations were with one processor). Comparison of the numerical results with measurements showed an overestimation of the spectral sound power at middle and high frequencies. Analysis of the intensity spectra in different points around the flame suggests that the effect of the inhomogeneity of the medium may not be negligible.

This work pretends to improve our hybrid method by considering sound propagation in inhomogeneous medium using the acoustic analogy but avoiding the direct evaluation of the three-dimensional volume integral. By open flames, the information about the "equivalent source terms" should be taken direct from the CFD calculation, while by enclosed flames, these sources may in some cases have to be modelled, since in some codes, no data is available outside the combustion chamber.

DESCRIPTION OF THE METHOD

We consider that all acoustic sources of the flame are located inside the control surface S_0 whose normal vector $\mathbf{n_0}$ is pointing to the outside and completely encloses the flame (see Fig. 1a). At the surface S_0, the velocity field is provided by the CFD calculation. Outside S_0, there is an inhomogeneous region in the space of volume Ω delimited by the surface S_1 (with normal vector $\mathbf{n_1}$ also pointing to the outside), whose density and sound velocity vary locally, $\rho(x)$, $c(x)$.

To determine the sound radiation, the space is divided in two regions (I and II) where the following differential equations have to be solved:

$$\begin{aligned} \left(\nabla^2 + k^2\right)p_I + D^{NL} &= 0 \quad \text{Region I} \\ \left(\nabla^2 + k_0^2\right)p_{II} &= 0 \quad \text{Region II} \end{aligned} \qquad \text{(Eq. 1)}$$

where k=ω/c is the wavelength and D^{NL} represents terms containing all non homogeneities. Since the sound speed in Ω is not constant, *k* depends on the position.
The boundary conditions at the interface between regions I and II demand continuity of pressure and particle velocity

$$\begin{aligned} p_I &= p_{II} \quad , \text{ on } S_1 \\ \frac{1}{\rho}\frac{\partial p_I}{\partial n_1} &= \frac{1}{\rho_0}\frac{\partial p_{II}}{\partial n_1} \quad , \text{ on } S_1 \end{aligned} \qquad \text{(Eq. 2)}$$

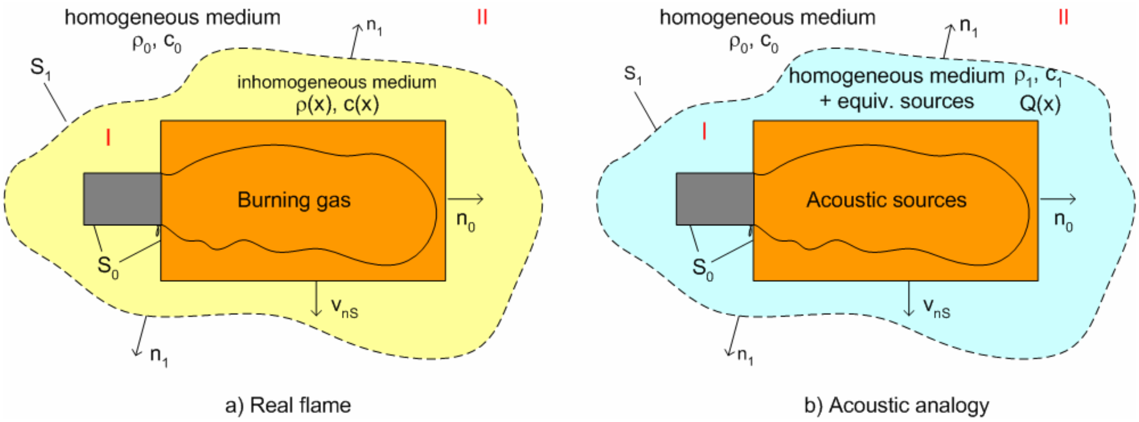

Figure 1: Description of the flame model

Following the original acoustic analogy, the differential equation in region I can be written as:

$$\left(\nabla^2 + k_1^2\right)p_I = -Q_\omega \qquad \text{(Eq. 3)}$$

with $Q_\omega = \left(k^2 - k_1^2\right)p_I + D^{NL}$ and k_1 a constant arbitrary wave number. We note, that the source term at the right hand side contains also the pressure (unless $k=k_1$), i.e. the pressure in Ω should be known. For the following derivations, Q_ω is considered to be given. Our new model has now a homogeneous medium surrounding the control surface S_0 and an additional source distribution Q_ω (see Fig. 1b).

Using the usual boundary element procedure, the differential equations in (1) and (3) are transformed into their integral form:

$$\begin{aligned} C_I p_I &= -\int_{S_1}\left(p_I^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_I^S}{\partial n_1} g_1\right)dS + \int_{S_0}\left(p_I^S \frac{\partial g_1}{\partial n_0} - \frac{\partial p_I^S}{\partial n_0} g_1\right)dS + \int_{\Omega} Q_\omega g_1 dV \quad \text{Region I} \\ C_{II} p_{II} &= \int_{S_1}\left(p_{II}^S \frac{\partial g_0}{\partial n_1} - \frac{\partial p_{II}^S}{\partial n_1} g_0\right)dS \quad \text{Region II} \end{aligned} \qquad \text{(Eq. 4)}$$

with

$$g_0 = \frac{e^{-jk_0R}}{4\pi R} \quad , \quad g_1 = \frac{e^{-jk_1R}}{4\pi R} \quad , \quad R = \left|\vec{x} - \vec{y}\right|$$

where $\vec{x}$ defines a field point and $\vec{y}$ a point at the surface, and the constants:

$$C_I = \begin{cases} 1 & \text{in } \Omega \\ 0.5 & \text{on } S_1 \\ 0 & \text{outside } \Omega \end{cases} \quad , \quad C_{II} = \begin{cases} 0 & \text{in } \Omega \\ 0.5 & \text{on } S_1 \\ 1 & \text{outside } \Omega \end{cases}$$

In Eq. (4), we can recognize the volume integral that increases the computational cost of the actual expression for the sound pressure.
According to the theory of differential equations, the general solution p_I can be written as the sum of a homogeneous and a particular solution of Eq (3): $p_I=p_h+p_u$, where p_h is the solution of the homogeneous equation and fulfil the boundary conditions, and p_u solves the inhomogeneous equation but does not fulfil the boundary conditions.

For the particular solution p_u, a relation similar to (4) applies:

$$Cp_u = -\int_{S_1} \left(p_u^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_u^S}{\partial n_1} g_1 \right) dS + \int_{S_0} \left(p_u^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_u^S}{\partial n_1} g_1 \right) dS + \int_{\Omega} Q_\omega g_1 dV \qquad \text{(Eq. 5)}$$

We can move the surface integrals of Eq. (5) to the left side and the volume integral $\int_{\Omega} Q_\omega g_1 dV$ can be written in terms of surface integrals.
Inserting (5) in (4), the new expression for the sound pressure in region I is given by:

$$\begin{aligned} C_I p_I = &-\int_{S_1} \left(p_I^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_I^S}{\partial n_1} g_1 \right) dS + \int_{S_0} \left(p_i^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_i^S}{\partial n_1} g_1 \right) dS + \\ & Cp_u + \int_{S_1} \left(p_u^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_u^S}{\partial n_1} g_1 \right) dS - \int_{S_0} \left(p_u^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_u^S}{\partial n_1} g_1 \right) dS \end{aligned} \qquad \text{(Eq. 6)}$$

Eq. (6) demonstrates that if a particular solution of the inhomogeneous differential equation is known, the pressure could be written in terms of surface integrals alone.

For most hybrid approaches, the source term Q_ω is known from the CFD calculations and not p_u. Hence, the particular solution has to be determined. A usual way to approximate a function is expanding it in a series of basis functions ψ_j

$$p_u(\vec{x}) = \sum_j \alpha_j \psi_j(\vec{x}) \qquad \text{(Eq. 7)}$$

When we replace (7) in (6), we obtain then:

$$\begin{aligned} C_I p_I = &-\int_{S_1} \left(p_I^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_I^S}{\partial n_1} g_1 \right) dS + \int_{S_0} \left(p_i^S \frac{\partial g_1}{\partial n_1} - \frac{\partial p_i^S}{\partial n_1} g_1 \right) dS + \\ & \sum_j \alpha_j \left(C\psi_j + \int_{S_1} \left(\psi_j^S \frac{\partial g_1}{\partial n_1} - \frac{\partial \psi_j^S}{\partial n_1} g_1 \right) dS - \int_{S_0} \left(\psi_j^S \frac{\partial g_1}{\partial n_1} - \frac{\partial \psi_j^S}{\partial n_1} g_1 \right) dS \right) \end{aligned} \qquad \text{(Eq. 8)}$$

where the coefficients α_j are still unknown.
Now we can use the fact that the source term Q_ω is given. Since p_u is a solution of the inhomogeneous equation, we can determine the values of α_j from the following relation:

$$-Q_\omega(\vec{x}) = \sum_j \alpha_j f_j(\vec{x}) \quad , \quad f_j(\vec{x}) = \left(\nabla^2 + k_1^2\right)\psi_j(\vec{x}) \qquad \text{(Eq. 9)}$$

and by discretizing the surfaces S_0 and S_1, a linear system of equations can be deduced.

The same relations (8) and (9) are derived under the concept of the Dual Reciprocity BEM in [3] but in reverse order, starting by fixing the functions f_j

The success of the method depends obviously on the set of functions ψ_j used and how good the source term Q is reproduced. From Eq. (9), it is clear that the set of basis functions can not be the solutions of the homogeneous Helmholtz equation.

NUMERICAL EXAMPLE

The accuracy of the method has been tested applying it to compute the sound radiation of a spherical flame [3]. The flame model consists of a spherical volume of hot gas with radius a, density ρ_1, sound speed c_1 and a sound source distribution $Q_\omega = Q(r)$, which is constant for all frequencies. The flame is surrounded by air with constants ρ_0 and c_0 (see Fig. 2a)

The analytical solution has the form:

$$\begin{aligned} p_I &= Q(Aj_0(k_1 r) - 1)/k_1^2 \qquad r \le a \\ p_{II} &= Te^{-jk_0 r}/r \qquad\qquad r \ge a \end{aligned} \qquad \text{(Eq. 10)}$$

and the constants A and T are determined from the boundary conditions in Eq. (2)

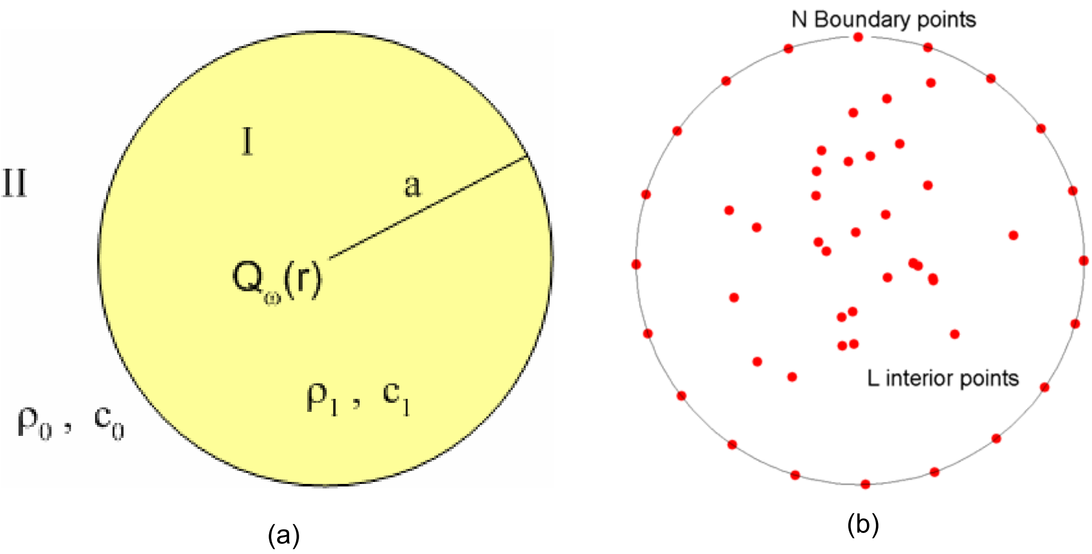

Figure 2: a) Spherical flame; b) Discretization points

The control surface S_1 in Eq. (6) is given by $r=a$ while there is no control surface S_0. The expanding functions chosen to define p_u where the same used in [5]:

$$\psi_j(\vec{x}) = \frac{1+r_j}{k^2} - \frac{2}{k^4}\left(\frac{1-\cos(kr_j)}{r_j}\right)_j \quad , \quad r_j = \left|\vec{x} - \vec{y}_j\right| \qquad \text{(Eq. 11)}$$

with corresponding functions:

$$f_j(\vec{x}) = 1 + r_j \qquad \text{(Eq. 12)}$$

The flame surface was represented by a sphere with 640 elements. For the determination of the coefficients α_j, besides the elements at the spherical surface, L=200 points in the interior of the sphere where taken (Fig. 2b), i.e, p_u was approximated with a total of 840 functions.

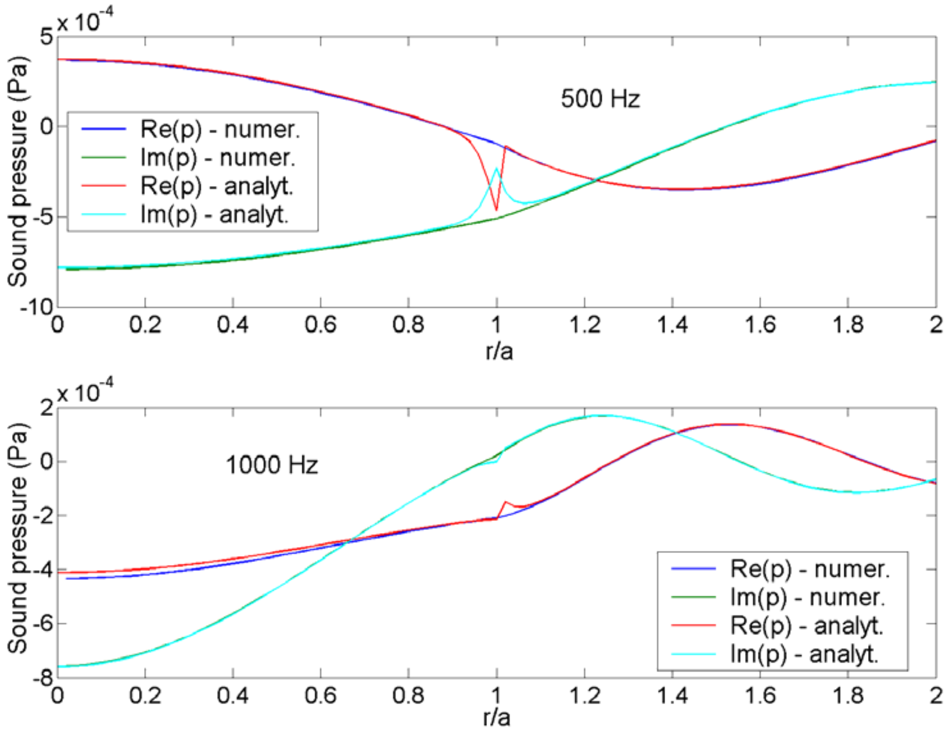

Figure 3: Comparison of the sound pressure at different positions

Fig. 3 shows a comparison of the analytical and theoretical values of the sound pressure. The agreement between analytical and numerical results is excellent. Only at the interface between Region I and II, the error of the numerical calculations is noticeable, but this error does not affect the sound power (see Fig. 4).

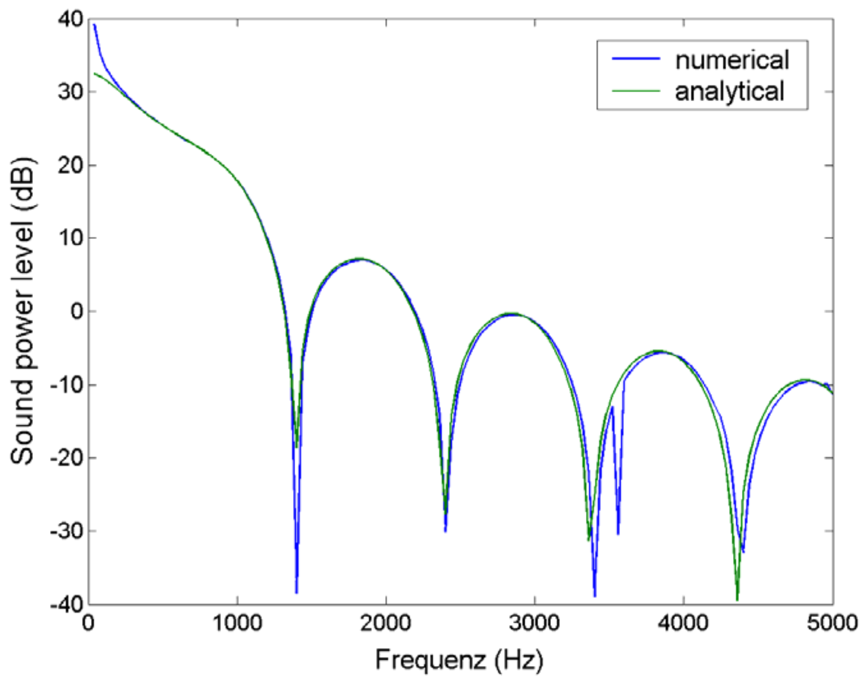

Figure 4: Sound power level in the frequency domain

SUMMARY

A method to calculate the sound radiation of flames considering the propagation in inhomogeneous medium based on an integral formulation has been presented. The velocity field at a control surface surrounding the flame, has to be previously determined (with a CFD code, for example) and the inhomogeneities of the medium have to be represented as source terms. The good agreement between numerical and analytical results obtained from a simple case encourages the application to more complex configurations.

AKNOWLEDGEMENTS

This work was supported by the German Research Foundation (DFG) within the Research Unit „Combustion Noise“.

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