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THE SOURCE SIMULATION TECHNIQUE WITH COMPLEX SOURCE POINTS FOR COMPUTING ACOUSTIC RADIATION PROBLEMS

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Abstract

The Source Simulation Technique is a general tool for calculating the sound radiation or scattering from complex-shaped structures into the three-dimensional space. The basic idea of the method consists in replacing the structure by a system of acoustical sources placed in the interior of the structure. By definition, these source functions have to satisfy the Helmholtz equation and the radiation condition. For solving the radiation or scattering problem completely or approximately, the source system also has to fulfil or minimize the boundary conditions on the surface of the body. In most cases, spherical wave functions e.g. monopoles, dipoles, quadrupoles etc. with different source locations are used as sources, since they can be calculated easily. If the coordinates of the source positions are shifted from real values into the complex plane, the corresponding monopoles show a similar behaviour like surface waves or possess a distinctive directivity pattern, depending on the values of the imaginary parts of their coordinates. Therefore, by adding such complex source point solutions to the system of equivalent sources, a strongly focused sound field should be computed in a more efficient and stable way. The capacity of the extended source simulation technique is investigated and represented by applying it to calculate the sound field radiated from a circular baffled piston.

INTRODUCTION

The basic building block of the Source Simulation Technique with complex source positions (also called **C**omplex **E**quivalent **S**ource **M**ethod with abbreviation CESM) is a monopole or point source, where the source position is not a real but a complex point in "space". Hence, we firstly investigate the properties of such a complex monopole. It will be demonstrated that this kind of sources behave in a completely

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different way as common real sources. For example, the singularity of a complex point source is not only a point, but is extended to an entire disc. Subsequently, the CESM is shortly described. Since the directivity pattern of a circular piston in a rigid baffle vibrating at high frequencies is strongly focussed and an analytical solution for the sound field is known, it is a good candidate for testing the CESM. A more detailed description of the theory and the numerical implementation of the CESM together with additional computational examples compared with calculations of the boundary element method can be found in [1] in the near future. Complex source positions can also be included in a BEM code to describe the effects of a ground with finite impedance [9].

MONOPOLE WITH A COMPLEX SOURCE POINT

A point source in the frequency domain at the source location $\vec{y} = (x_s, y_s, z_s)$ in three-dimensional space is given by

$$g(\vec{x},\vec{y}) = A\exp(-jkR)/(4\pi R) \quad , \quad R = \sqrt{(x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2} \; , \qquad (1)$$

where A is the complex amplitude, $\vec{x} = (x, y, z)$ is the receiver position, $\vec{y} = (x_s, y_s, z_s)$ is the source position, $k = \omega/c$ is the wave number with angular frequency ω, and c is the speed of sound. All time-varying quantities should obey the time dependence $\exp(j\omega t)$ with $j = \sqrt{-1}$. $g(\vec{x},\vec{y})$ is the free-space Green's function and solves the inhomogeneous Helmholtz equation

$$\Delta g + k^2 g = -\delta(\vec{x},\vec{y}), \qquad (2)$$

where Δ is the Laplacian, and δ is the Dirac delta function. Now, we introduce a complex source point by adding imaginary parts to the real source coordinates:

$$\vec{y}_c = (x_S - ja, y_S - jb, z_S - jc) = \vec{y} - j\vec{\beta} \quad , \quad \vec{\beta} = (a,b,c) \qquad (3)$$

Such a monopole with a complex source point is also a solution of the Helmholtz equation with respect to the spatial coordinates $\vec{x} = (x, y, z)$ for a fixed complex source position $\vec{y}_c$. The proof can be found in [2]. However, it must be taken into account that the singularities of the complex source are different from the point singularity $\delta(\vec{x},\vec{y})$ of the ordinary monopole. The complex distance

$$R = R_r + jR_i$$

becomes zero on the circle

$$C = \left\{ \vec{x} \text{ in } E \,/\, |\vec{x} - \vec{y}| = \beta \right\} \; , \; \beta = |\vec{\beta}|$$

over the plane E , $E = \left\{ \vec{x} \,/\, (\vec{x} - \vec{y}) \cdot \vec{\beta} = 0 \right\}$ and with centre in $\vec{y} = (x_s, y_s, z_s)$. This is shown in Fig. 1.

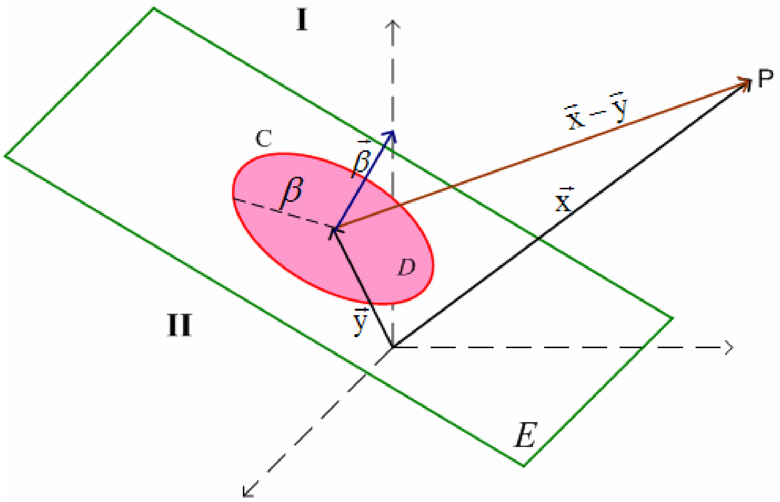

Figure 1: Complex source point and its geometric features.

Furthermore, the complex root must be made single-valued. It can be shown (see Kaiser [3]) that a suitable branch cut, requiring $\mathrm{Re}\{R\} = R_r \geq 0$ for satisfying the radiation condition, is given by the disc

$$D = \left\{ \vec{x} \text{ in } E \,/\, |\vec{x} - \vec{y}| \leq \beta \right\}.$$

Hence, the complex source becomes singular on the source disc D. Outside of D, the complex source is an exact solution of the Helmholtz equation. The disc D determines a plane, which divides the space in two regions. Above the plane (region I), the amplitude of the wave is amplified by e^{kR_i} while below the plane (region II), the wave is attenuated by $e^{-k|R_i|}$.

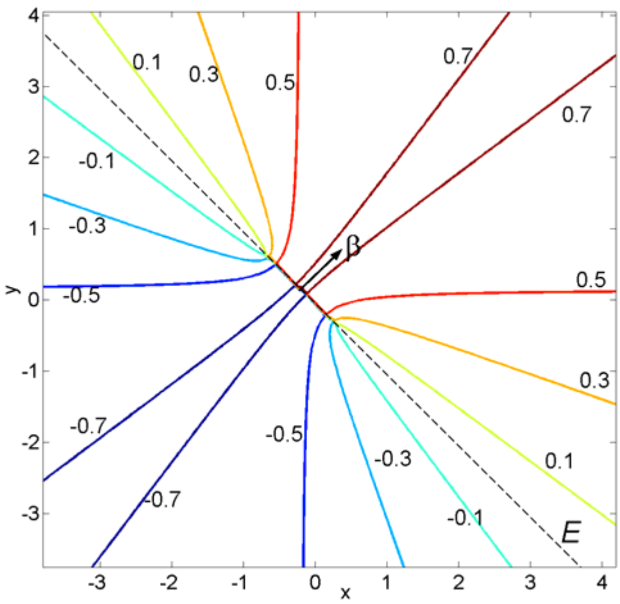

Figure 2: Values of R_i for $\vec{y} = (-0.2, 0.15, 0)$ *and* $\vec{\beta} = (0.5, 0.5, 0)$, $\beta = 0.7071$

The values of R_i are limited to the interval $[-\beta, \beta]$, and $R_i = \beta$ occurs at points on a line spanned by $\vec{\beta}$ and passing through $\vec{y}$ which we call $\vec{\beta}$-axis. In Fig. 2, contours of constant R_i at the plane XY for a given $\vec{\beta}$ are shown. Notice that they are symmetric with respect to the $\vec{\beta}$-axis and positive above *E* and negative below *E.*

The values of R_i together with the factor e^{kR_i} cause the sound to be radiated mainly in a half space and concentrated in the paraxial region near the axis given by $\vec{\beta}$, when the magnitude of $\vec{\beta}$ or the wave number *k* is not too small. This is in contrast to the uniformly radiation into the whole space of the usual monopole (see Fig. 3).

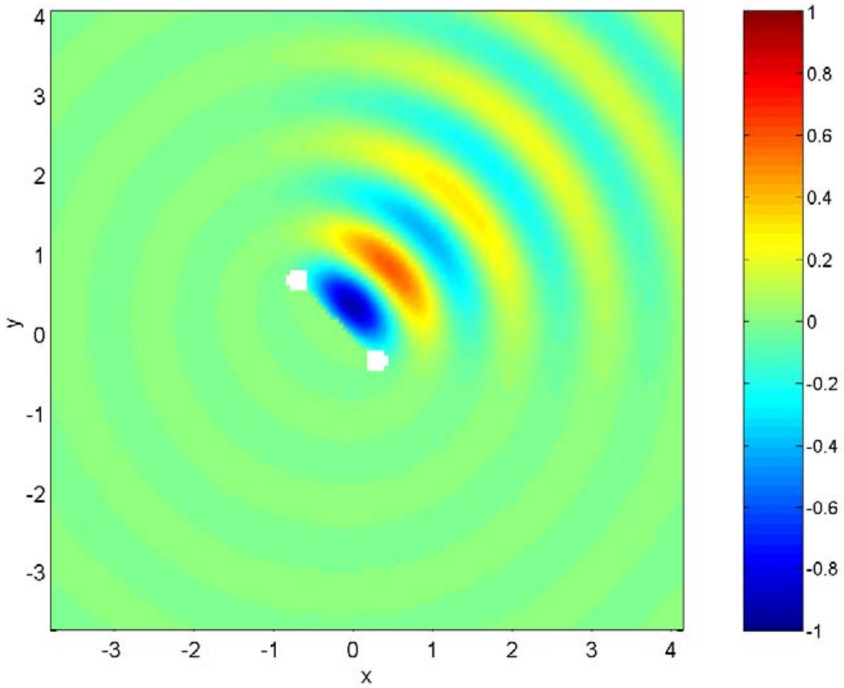

Figure 3: Sound radiation of a monopole with a complex source position with the parameters given in Fig. 2 and a frequency of 300 Hz

The small white circles that appear in Figs 2 and 3 are the intersections of the singular circle C with the plane XY. More details about complex sources, the related literature and its application to acoustical half-space problems can be found in [4,5]

THE COMPLEX EQUIVALENT SOURCE METHOD

Theory and numerical applications of the Source Simulation Technique, also called the Equivalent Source Method (ESM) can be found in full detail in [6,7]. In its traditional form, the ESM only uses sources (for example: monopoles, dipoles quadrupoles), which have real source positions (shortcut: real sources).

Now, the ESM is extended by adding sources with complex source positions (shortcut: complex sources) as given in Eq. (3) to the source system. We denote such an enhanced method as complex equivalent source method and use the abbreviation CESM. Thus, the CESM replaces the vibrating structure by a system of real and complex sources in such a way, that the sound field radiated by the structure or by the source system is (approximately) the same. Since the source system fulfils the Helmholtz equation and the radiation condition automatically, it is only necessary that the source system satisfies the boundary condition at the structural surface S, in order to obtain an accurate solution of the radiation problem. For example, if the normal velocity v_{nS} is prescribed at *S*, then the relative quadratic surface error (F_{rel})

$$F_{rel} = \frac{\int_S \left| v - v_{nS} \right|^2 dS}{\int_S \left| v_{nS} \right|^2 dS} \tag{4}$$

is a measure for the quality of the solution, where *v* is the velocity due to the equivalent sources (see [4,5]). In addition, the condition number (κ)

$$\kappa = \left\| A \right\| \left\| A^{-1} \right\| \quad \left(\| \cdot \| = \text{norm} \right) \tag{5}$$

is an important parameter for the stability of corresponding numerical method, where *A* is the matrix of the system of linear equations resulting from the discretization of the ESM or CESM. Unfortunately, the condition number can become very high, so that it is often necessary to regularize the method by using the singular value decomposition or a certain minimising technique like the Full-Field-Equation proposed in [8].

NUMERICAL EXAMPLE: RADIATION OF A CIRCULAR BAFFLED PISTON

We investigate the sound radiation from a circular piston in an infinite rigid baffle, which is vibrating in phase with the normal velocity v_0. At high frequencies the piston radiates sound strongly focussed into the direction of its axis, and hence such a radiator is a well suited object for testing the ability of the CESM to simulate such directivity patterns. Only at the axis through the piston, an analytical solution for the sound pressure can be obtained by

$$p_{axis}(x) = \rho c v_0 \left(e^{-jk\sqrt{R_p{}^2 + x^2}} - e^{-jkx} \right), \tag{6}$$

where R_p is the radius of the piston and v_0 its normal constant velocity.
In the far-field ($kr >> 1$), an approximate solution is given by

$$p_{far}(r,\theta) = \frac{j\rho c k^2 R_p{}^2 v_0 e^{-jkr}}{2kr} \left[\frac{2J_1(kR_p \sin\theta)}{kR_p \sin\theta} \right] \tag{7}$$

where θ is the angle between the field point $\vec{r}$ and the axis of radiation.

For the numerical calculations, a circular baffle with radius $R_p = 0.1\text{m}$ was imbedded in a square plate with a side length of 2 m. The elements are squares with a side length of 0.0167 m, so that six points per wavelength up to 3430 Hz are ensured. However, we have used this model for a higher frequency, 6860 Hz, and good results were still obtained. The total number of elements is 14.400. In order to simulate a circular piston, the elements whose centres lie within an area with radius 0.1 m are assumed to vibrate with the constant normal velocity amplitude of 1 m/s (see Fig. 4a).

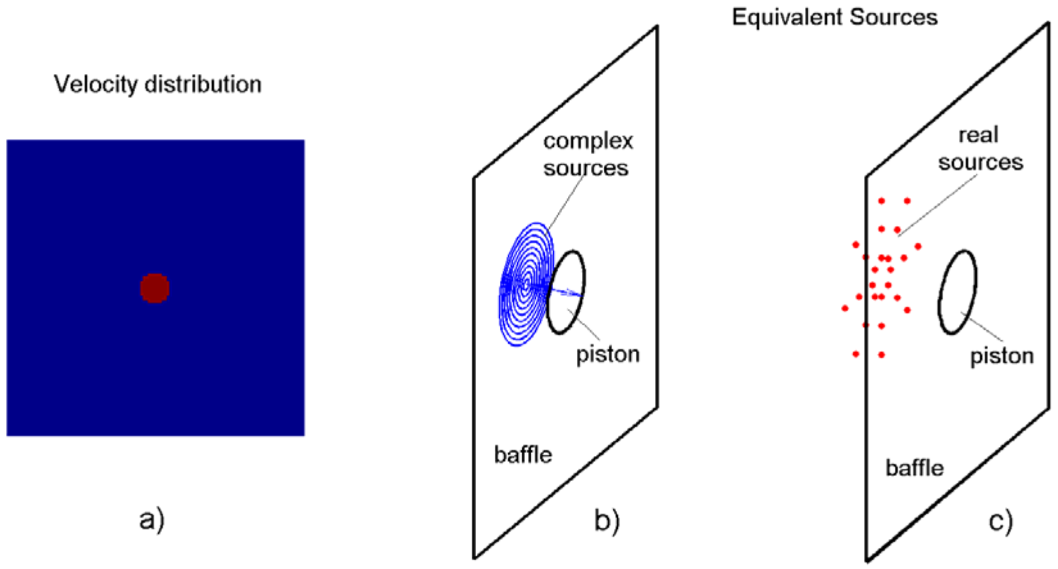

Figure 4: Velocity distribution a) and equivalent source positions b) complex; c) real

A series of calculations shows that an appropriate source system consists of ten complex monopoles, which lie directly behind the piston in a distance of 0.005 m (see Fig. 4b). The x-component of the source positions possesses a growing imaginary part which varies from 0.015 up to 0.15. To appreciate the advantage of using complex sources a calculation with the normal ESM was made too (Fig. 4c). A comparison of the parameters from both calculations is presented in Table 1.

Source Positions	N° of sources	F_{rel} (%)		κ		Calculation time (s)
		3430 Hz	6860Hz	3430 Hz	6860Hz	
complex	10	4.8	3.3	7.9×10^{6}	2.0×10^{15}	13
real	216	5.2	6.0	1.9×10^{13}	1.2×10^{8}	300

Table 1: Comparison of the calculation parameters of the ESM and CESM

One can notice that to achieve a similar degree of accuracy, only ten complex monopoles are needed in contrast to 216 real multipoles (monopoles, dipoles and quadrupoles). This decreases the time of calculation on an ordinary PC from 300 s to 13 s, i.e. more than 20 times. Another feature of the complex source system is that the condition number κ increases with the frequency contrary to the case of real sources where κ decreases with the frequency.

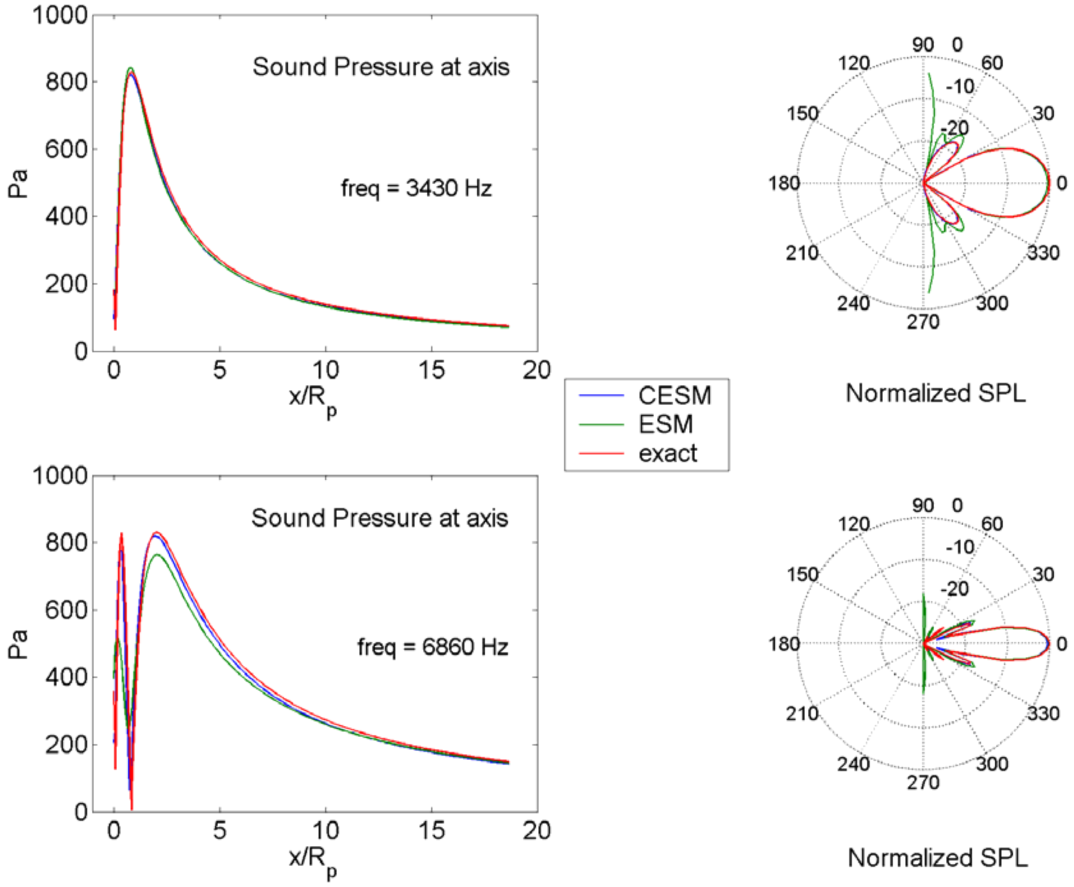

Figure 5: Comparison of the analytical sound pressure with the results of the CESM and ESM at the axis (left) and in the far-field (right).

In Fig. 5, the exact sound pressure at the axis (see Eq. (6)) and at a circle of radius 100 m (see Eq. (7)) is compared with the sound pressure calculated with the complex and real source systems for the two studied frequencies. Since the values of the pressure are complex, the absolute values are presented. The radiation patterns at the right are normalized with respect to the value at 0°. As it can be easily seen, the agreement obtained with the complex sources is excellent.

Though the surface error F_{rel} obtained by the real sources is comparable to the surface error of the complex sources, the error of the sound pressure at the axis at 6840 Hz is slightly bigger with the ESM than with the CESM. Also, the radiation patterns of the ESM present a false radiation in the plane of the baffle, which indicates that this type of radiator is difficult to handle with the normal ESM.

SUMMARY

By adding monopoles with complex source points to the source system of the Equivalent Source Method, strongly focused sound fields similar to sound beams can be numerically simulated with promising accuracy. Also, the corresponding computing time decreases significantly. Hence, the CESM could be used for such radiator configurations with advantage. Work is in progress [1] to apply the CESM to real-life radiators with more complicated surface geometry.

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