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NUMERICAL ASPECTS OF THE EQUIVALENT SOURCE METHOD APPLIED TO COMBUSTION NOISE

R. Piscoya[1,2], H. Brick[1,2], M. Ochmann[1], P. Költzsch[2]

[1] Fachbereich II, Mathematik, Physik, Chemie
Technische Fachhochschule, Berlin
Luxemburger Str. 10, 13353 Berlin, Germany
piscoya@tfh-berlin.de

[2] Institut für Akustik und Sprachkommunikation,
Technische Universität Dresden
01062 Dresden, Germany

Abstract

The use of the Equivalent Source Method (ESM) to compute the sound field originated by combustion is one of the subjects that are being investigated by the research group “Combustion Noise”. The ESM has been coupled to a LES code in order to calculate the sound radiation of open diffusion flames. The aim of this paper is basically to discuss numerical aspects of the ESM that must be considered for an appropriate computation of the sound fields. The number, position and order of the equivalent multipoles are free parameters that have to be fixed before the amplitudes of the sources are found. These parameters have an important influence on the accuracy of the results. Different distributions of multipoles and a distribution of monopoles alone, whose positions were optimized by a simple engineering procedure, are tested and the sound fields are compared. Two quantities are usually used to give a measure of the accuracy of the method: the error at the surface and the condition of the matrices. These quantities are computed for every group of sources and compared, too. An additional processing of the data at the control surface to reduce the variance in the power spectrum is shown. To validate the accuracy of the ESM calculations, the sound radiation of the flame was computed with a standard BEM program using the same acoustic mesh and boundary conditions and the results are compared with those of the ESM.

INTRODUCTION

The Equivalent Source Method (ESM) is widely used for the calculation of the sound field radiated or scattered from complex-shaped structures. Extending the applicability of this method for the prediction of the sound radiated from aero- and thermo-acoustical sources, is one of the subjects studied by the research group "Combustion Noise". A general purpose of this group is the development of a toolbox for determining the sound produced by open and closed flames, covering from the source simulation up to the calculation of the far field [1]. The ESM has been used to compute the radiated sound in the far field produced by open flames from information of the near field obtained by a Large Eddy Simulation (LES) [2], [3]. The coupling of these two methods, LES and ESM, results in a so called hybrid method.

The principle of the ESM is to replace the original sound source with a system of equivalent multipoles whose amplitudes are determined in a way that a boundary condition is satisfied [4]. The location and nature of the real sound sources inside the flame remain a matter under investigation. In this respect, the ESM could be used to take a look at the mechanisms of sound generation inside the flame by comparing the position and amplitudes of the equivalent sources with physical variables like the heat release variations and the fluctuations of local volume and density. A first step in this direction is a study of the influence of the position, number and order of the equivalent multipoles on the predicted sound field. The accuracy of the results of the ESM depends on how good the boundary condition is fulfilled but also on the stability of the system of equations.

This paper starts with a brief description of the processing of the data from the LES, in particular the calculation of the velocity spectra. Then, different equivalent source distributions are tested and the quality of their results is compared by an evaluation of some parameters defined for that purpose. The results of the ESM are compared with those of a BEM calculation that uses the same control surface and boundary conditions.

PROCESSING OF THE LES DATA

For the acoustic calculations, the open flame is modelled as a radiating cylinder, whose surface vibrates with a normal velocity equal to the local normal velocity of the fluid. For the calculations, the cylindrical surface is discretized in a certain number of elements. The length and radius of the cylinder as well as the size of the elements are determined according to the LES geometric parameters. The velocity field is calculated in the time domain by the LES. Since the developed ESM program works in the frequency domain, the velocity has to be Fourier transformed. The spectral components obtained from the Fourier Transform have a large variance due to the stochastic nature of the signal. In order to reduce this variance, the time signal is broken up into several segments with 50% overlap. Each segment is windowed and Fourier transformed. A sound field is evaluated using the ESM with each one of the resulting spectra and the estimated sound field is the average over all calculations.

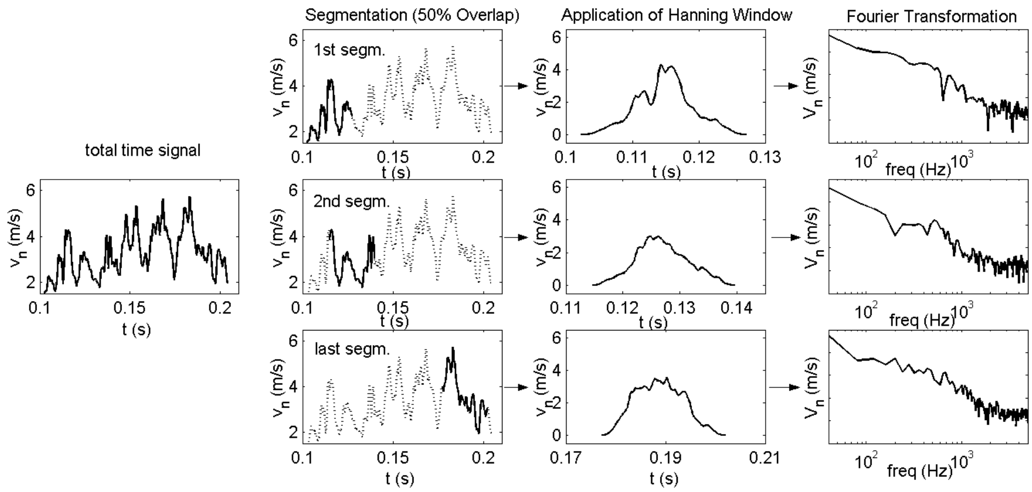

Figure 1: Schema of the computation of the velocity spectra.

COMPUTATION OF THE EQUIVALENT SOURCES

The equivalent sources are described by spherical wave functions, which are solutions of the Helmholtz equation and satisfy the Sommerfeld radiation condition at infinity. The most general expression for the sound pressure generated by Q multipole sources with order up to N, that are located at positions given by the vectors $\vec{r}_q$, is:

$$p(\vec{r}) = \sum_{q=1}^{Q}\sum_{n=0}^{N}\sum_{m=0}^{n}\frac{h_n^{(2)}(k|\vec{r}-\vec{r}_q|)}{h_n^{(2)}(kR)}P_n^m(\cos\gamma_q)\left(A_{nm}^{(q)}\cos(m\delta_q)+B_{nm}^{(q)}\sin(m\delta_q)\right) \quad (1)$$

where P_n^m are the Legendre functions, $h_n^{(2)}$ are spherical Hankel functions of the second kind and γ_q and δ_q are the angles between the vector $(\vec{r}-\vec{r}_q)$ and the Z and X axis respectively. The spherical wave functions in equation (1) have been normalized using $h_n^{(2)}(kR)$, where R is the radius of the cylindrical surface. The total number of spherical functions and thus the number of equivalent sources in expression (1) is $N_{tot}=Q\,(N+1)^2$. For simplicity, equation (1) is written as:

$$p(\vec{r}) = \sum_{i=1}^{N_{tot}} c_i\psi_i(\vec{r}) \quad (2)$$

where the subindex i represents a combination of q,n and m, and c_i and ψ_i are the amplitude and wave function for that q,n,m combination. The amplitudes c_i are determined by minimizing the velocity error at the control surface using the method of weighted residuals [4]. The weighting functions used for the calculations are the complex conjugate normal derivatives of the wave functions $w_l = \partial\psi_l^*/\partial n$. The selection of these functions implies a minimization of the velocity error in the least square sense. With the discretization of the surface and assuming constant values over each element, equation (4) is transformed into a system of linear equations ***Ax=b*** that has to be solved. ***A*** is a $N_{tot} \times N_{tot}$ matrix and ***x*** is the vector of amplitudes c_i.

COMPARISON OF EQUIVALENT SOURCE DISTRIBUTIONS

In order to compare the accuracy of the results coming from different equivalent sources distributions, four parameters are introduced:

1) The relative surface error (F_{rel}) estimates how good the velocity at the control surface (v_{nS}) has been reproduced by the simulated velocity (v). A small value of F_{rel} indicates that the boundary condition has been well satisfied and the sound field will be accurately computed. Strong near fields may produce large surface errors, but the radiated sound can still be reasonably well approximated.
2) The condition number (κ) is a real number that measures the sensitivity of the solution ***x*** to perturbations in ***b***. The condition number evaluated in the calculations is the ratio of the largest to the smallest singular value of ***A***. The condition number of stable systems is one, while bad conditioned systems have large condition numbers.
3) The field error (E_f) gives a measure of the difference in sound pressure level over a collection of field points, respect to reference values. In our case, 360 points over a sphere of radius 1m have been taken to evaluate E_f. The reference values are obtained by a BEM calculation using the same control surface and boundary condition as the ESM.
4) The sound power error (E_W) is the difference in the sound power level respect to a reference value. For the studied flames, the reference value is again obtained from the BEM calculation.

Relative surface error ($\boldsymbol{F_{rel}}$)	Condition number ($\boldsymbol{\kappa}$)	Sound field error ($\boldsymbol{E_f}$)	Sound power level error ($\boldsymbol{E_W}$)
$\dfrac{\int_S \lvert v - v_{nS}\rvert^2 dS}{\int_S \lvert v_{nS}\rvert^2 dS}$	$\lVert A \rVert \lVert A^{-1} \rVert$ ($\lVert \cdot \rVert = \text{norm}$)	$median\left[\left\lvert 20\log_{10}\left(\dfrac{\lvert p(x)\rvert}{\lvert p_{ref}(x)\rvert}\right)\right\rvert\right]$	$L_W^{(ESM)} - L_W^{(ref)}$

Table 1: Definition of the parameters used to compare the equivalent source distributions.

Results

The configuration of the studied flames suggests that the generated sound field should be symmetric with respect to the flame axis. This fact has been proven by acoustic measurements. For this reason all distributions of sound sources except one show this symmetry. The axis of the cylindrical control surface coincides with the flame axis. In the first case one multipole has been located at the axis and the order has been varied from 5 to 11. To see the influence of the position of the source, 5 different positions were tested. In the second case the same five positions at the axis are filled with multipoles. The order of the multipoles varies from 2 to 8. In the third case, the order of the multipole has been set to 2 and three different distributions have been tested: a) at the flame axis, b) over parallel rings centered at the axis and c) at random positions. Finally, in the last case, only monopoles have been used to replace the flame, since qualitative analysis of the governing equations indicate that the main sound sources would have a monopole nature [5]. The results of monopoles at axial, ring and random distributions are compared with the results of distributions of monopoles that result from a simple engineering procedure to optimize the source positions. In all cases, the four previously mentioned parameters are computed for 125, 250, 500, 1000 and 2000 Hz.

One multipole at the axis

The results show that the position of the multipole has a larger influence than its order. The multipole placed near the upper cap of the cylinder (P5), where the velocity is much greater, gives smaller values of the errors than the multipole placed in the middle of the cylinder (P1), however, these values are high. The surface error decreases with increasing order (5 to 9) but begin to grow at larger orders, because the condition number becomes bigger than the computer precision, which is about 10^{16}. The values of the compared parameters for points P1 and P5 at 500 Hz are listed in Table 1. From those values, it can be derived that using only one multipole does not give good results unless it is placed in a proper position.

	F_{rel} (%)		κ		E_f(dB)		E_W (dB)	
order	P1	P5	P1	P5	P1	P5	P1	P5
5	99.3	63.8	10^{6}	10^{7}	15.7	3.4	-8.4	-3.0
6	99.1	60.4	10^{9}	10^{9}	13.3	3.8	-8.0	-3.2
7	98.7	58.2	10^{11}	10^{12}	12.8	3.9	-6.8	-3.5
8	98.3	56.4	10^{14}	10^{15}	9.5	4.3	-6.2	-3.4
9	97.4	54.7	10^{17}	10^{18}	8.9	6.0	-4.9	-5.7
10	100.7	197.5	10^{20}	10^{21}	2.5	18.8	1.0	16.8
11	211.7	205.5	10^{22}	10^{24}	18.6	19.2	16.2	17.2

Table 1: Comparison of the parameters by one multipole at the axis

Five multipoles at the axis

The five previous positions are now occupied with multipoles. The values of the four parameters for 125 Hz and 500 Hz appear in Table 2. The results obtained with orders larger than 5 are not accurate, because of the high condition numbers of the matrices. The surface error diminishes with increasing order, but the field error and the power error do not change significantly. The crosses indicate that the simulation gives a negative sound power. Field errors and power errors are larger at 500 Hz than at the other frequencies (see Fig. 4). The results taking five multipoles are better or have at least the same accuracy than those of one multipole.

	F_{rel} (%)		κ		E_f (dB)		E_W (dB)	
order	125Hz	500Hz	125Hz	500Hz	125Hz	500Hz	125Hz	500Hz
2	82.3	71.2	10^{9}	10^{9}	2.1	3.7	-0.8	-3.0
3	71.7	66.6	10^{14}	10^{14}	1.2	3.8	-0.7	-3.3
4	61.4	60.6	10^{19}	10^{19}	0.6	4.3	-0.8	-3.4
5	51.0	58.5	10^{22}	10^{22}	0.5	3.8	-0.6	-3.0
6	41.5	55.5	10^{24}	10^{25}	2.5	3.0	x	-2.7
7	34.6	54.5	10^{26}	10^{27}	6.4	4.2	x	-2.3
8	914.1	116.6	10^{30}	10^{29}	35.2	22.3	x	-4.5

Table 2: Comparison of the parameters using five multipoles at the axis

Line, ring and random distributions

In Table 3, the parameters for the three source distributions are shown. The condition number of the ring distribution is several orders of magnitude smaller than the line and random distributions, probably because the sources are not too close from each other. The field and power errors of this distribution are slightly better than those of the other two source positions.

	F_{rel} (%)		κ		E_f (dB)		E_W (dB)	
distrib	125Hz	500Hz	125Hz	500Hz	125Hz	500Hz	125Hz	500Hz
Line (L)	59.3	60.0	10^{17}	10^{19}	0.6	4.2	-0.8	-3.3
Ring (Rg)	68.7	68.8	10^{11}	10^{11}	1.4	3.4	-0.4	-2.8
Rand (Rd)	81.7	69.4	10^{18}	10^{18}	2.1	3.5	-0.8	-3.0

Table 3: Comparison of the parameters by three source distributions

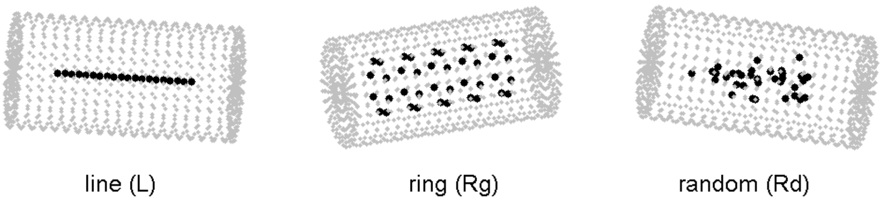

Figure 2: Distributions of equivalent sources

Only monopoles

Different number and distributions of monopoles have been tested. The results of the three previous distributions (L, Rg and Rd), but filled only by monopoles, are compared to those obtained with monopoles placed at "optimized" (O) positions. A group of 20, 40,…,120 positions are selected from 480 potential source positions following a procedure proposed in [6]. The line distribution provides the better results when using multipoles, compared to Rg and Rd. When using optimized positions, the surface error decreases with increasing number of monopoles but this decrease tends to stabilize for more than 80 sources. The value of F_{rel} depends on the frequency, at 125 Hz, it decreases to near 20% while at 500 Hz to 50%

	F_{rel} (%)		κ		E_f (dB)		E_W (dB)	
No. monop.	125Hz	500Hz	125Hz	500Hz	125Hz	500Hz	125Hz	500Hz
20 (L)	87.6	73.1	10^{15}	10^{15}	2.0	4.0	-0.9	-3.2
40 (Rg)	99.4	99.3	10^{3}	10^{3}	1.0	14.6	-1.1	-8.4
35 (Rd)	97.0	96.5	10^{7}	10^{7}	2.2	6.4	-0.6	-4.2
20 (O)	72.0	65.0	10^{2}	10^{2}	4.0	4.3	-1.6	-3.4
40 (O)	52.0	60.0	10^{4}	10^{4}	0.4	3.9	-0.4	-3.1
60 (O)	37.5	58.2	10^{5}	10^{4}	0.4	3.9	-0.8	-3.1
80 (O)	27.9	55.2	10^{6}	10^{5}	0.6	3.8	-0.4	-3.1
100 (O)	26.5	52.4	10^{6}	10^{6}	0.6	3.8	-0.5	-3.0
120 (O)	25.5	50.9	10^{7}	10^{6}	0.6	3.7	-0.5	-3.0

Table 4: Comparison of the parameters by different monopoles distributions

The first 80 selected positions (of the 480 possible positions) at 500 Hz are shown in Fig. 3. Most of them are located near the upper cap of the cylinder, where the normal velocity has its largest value as seen in the same figure. From Table 4, one notices that the field and power errors do not change when the number of monopoles are increased above 80, only the surface error becomes slightly smaller. In Fig. 4 the sound power level and radiation pattern as well as the surface error from the different distributions and from the BEM are shown. Despite the differences in the surface error, the sound power level is almost the same for all distributions and its error is often smaller than the field error.

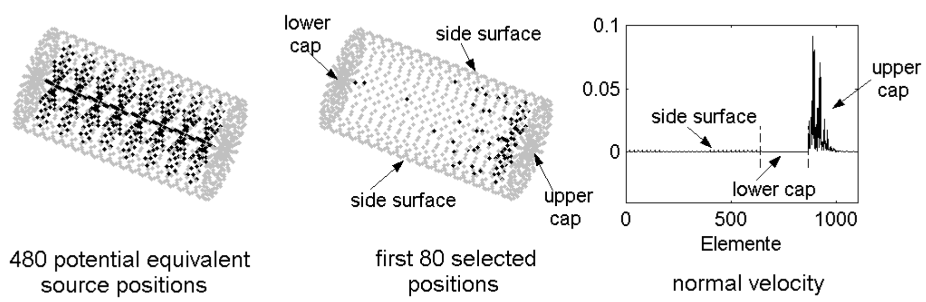

Figure 3: Optimized positions of the monopoles

CONCLUSIONS

The study and comparison of different equivalent sources distributions have shown that for our particular problem, the position of the multipoles have a stronger influence on the results as their orders. Also, the sound power level is less sensible to the equivalent source distributions than the sound field, especially at the surface boundary. This means that one can work with low order multipoles (up to 2), regularly distributed inside the control surface and expect an accurate prediction of the radiated sound. Using only monopoles, placed in “optimized” positions, have provided lower values of the surface error but have not reduce substantially the field and power errors. More tests, using optimization procedures could lead to a better correspondence between real and equivalent sources. Those procedures imply, however, an increase in the computation time, especially when several spectra have to be calculated and averaged.

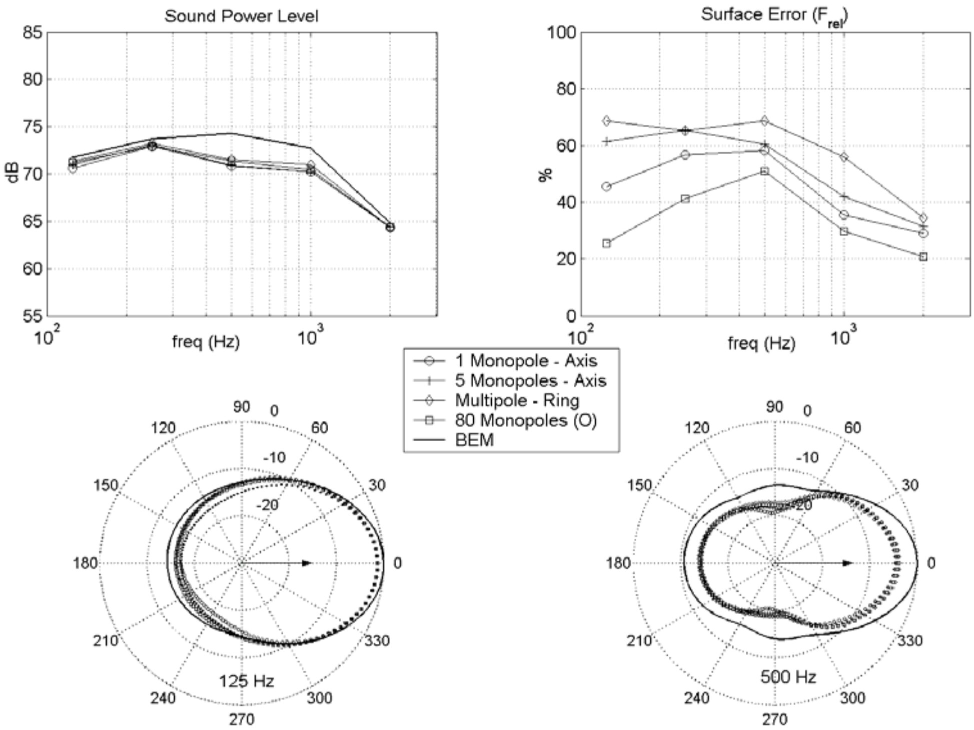

Figure 4: Comparison of sound power level, surface errors and radiation patterns.

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